Object A is moving due east, while object B is moving due north. They collide and stick together in a completely inelastic collision. Momentum is conserved. Object A has a mass of mA = 16.1 kg and an initial velocity of = 8.11 m/s, due east. Object B, however, has a mass of mB = 29.8 kg and an initial velocity of = 5.46 m/s, due north. Find the (a) magnitude and (b) direction of the total momentum of the two-object system after the collision.

Solution

Momentum is conserved. That's why

$$\overrightarrow{P_A} + \overrightarrow{P_A} = \overrightarrow{P_{A+B}},$$

where $\overrightarrow{P_A}$ is momentum of object A, $\overrightarrow{P_B}$ is momentum of object B, $\overrightarrow{P_{A+B}}$ is momentum of the two-object system after the collision.

Break each vector down into its horizontal and vertical components (y – due to north, x – due to east):

$\overrightarrow{P_A}$ is $(m_A v_A, 0)$, $\overrightarrow{P_B}$ is $(0, m_B v_B)$, $\overrightarrow{P_{A+B}}$ is $(m_A v_A, m_B v_B)$.

The magnitude of the total momentum of the two-object system after the collision:

$$\left|\overrightarrow{P_{A+B}}\right| = \sqrt{(m_A v_A)^2 + (m_B v_B)^2} = \sqrt{(16.1 * 8.11)^2 + (29.8 * 5.46)^2} = 209\ kg * \frac{m}{s}.$$

The direction of the total momentum of the two-object system after the collision:

$$\theta = \tan^{-1}\left(\frac{m_B v_B}{m_A v_A}\right) = \tan^{-1}\left(\frac{29.8 * 5.46}{16.1 * 8.11}\right) = 51° \text{ north of east.}$$

Answer: $209\ kg * \frac{m}{s}$ at $51°$ north of east.