1. A motor boat covers the distance between two spots on the river in t_1=8 hr and t_2=12 hr downstream and upstream respectively. The time required for the boat to cover this distance in still water will be…

$t_1 = 8\,hr$
$t_2 = 12\,hr$
$t_0 - ?$

Solution.

Let denote the speed of the boat in still water by v and the speed of the water by v_0. The speed of boat downstream and upstream is $v + v_0$, $v - v_0$, respectively.

Let the distance between the spots be equal to l.

The time, which is spent for covering this distance downstream and upstream respectively:

$$t_1 = \frac{l}{v + v_0}, \quad t_2 = \frac{l}{v - v_0}. \tag{1}$$

The time required for the boat to cover the distance in still water is

$$t_0 = \frac{l}{v}, \tag{2}$$

but we do not know both quantities l and v. So, we have to express the ratio $\frac{l}{v}$ from the system of the equations (1). Let do some transformations with these equations.

$$t_1 = \frac{1}{\frac{v}{l} + \frac{v_0}{l}}, \quad t_2 = \frac{1}{\frac{v}{l} - \frac{v_0}{l}},$$

$$\frac{v}{l} + \frac{v_0}{l} = \frac{1}{t_1}, \quad \frac{v}{l} - \frac{v_0}{l} = \frac{1}{t_2}.$$

Let write the sum of the last two expressions:

$$\frac{2v}{l} = \frac{1}{t_1} + \frac{1}{t_2}. \tag{3}$$

Dividing by 2, we obtain the required ratio:

$$\frac{v}{l} = \frac{1}{2}\left(\frac{1}{t_1} + \frac{1}{t_2}\right).$$

So, the time required for the boat to cover this distance in still water is

$$t_0 = \frac{l}{v} = \frac{1}{\frac{v}{l}} = \frac{1}{\frac{1}{2}\left(\frac{1}{t_1} + \frac{1}{t_2}\right)}, \quad \boxed{t_0 = \frac{2}{\frac{1}{t_1} + \frac{1}{t_2}}}.$$

Let check the dimension.

$$[t_0] = \frac{1}{\frac{1}{hr}} = hr.$$

Let evaluate the quantity.

$$t_0 = \frac{2}{\frac{1}{8} + \frac{1}{12}} = 9.6(hr), \quad \text{so } t_0 = 9.6\,hr\,36\,\text{min}.$$

Answer: $9.6\,hr\,36\,\text{min}$.