an engine working on Carnot cycle absorves heat from 3 blocks at 1000 kelvin, 800 kelvin and 600 kelvin. The engine develops 600 kj per min of work and reject 400 kj per min of heat to the block at 300 kelvin. If the heat supplied to the block at 1000 kelvin is 60% of the heat supplied by the block at 600 kelvin. Find the quantity of heat absorves by 3 blocks.

Solution

We define several quantities for a cycle:

Q_{A1} is the heat absorbed by block at $T_1 = 1000\ kelvin$,

Q_{A2} is the heat absorbed by block at $T_2 = 800\ kelvin$,

Q_{A3} is the heat absorbed by block at $T_3 = 600\ kelvin$,

Q_R is the heat rejected by the system,

W is the net work done by the system,

$T_4 = 300\ kelvin.$

The thermal efficiencies of the cycles:

$$\eta_1 = \frac{W_1}{Q_{A1}}, \eta_2 = \frac{W_2}{Q_{A2}}, \eta_3 = \frac{W_3}{Q_{A3}},$$

where W_1, W_2, W_3 - work done by 3 blocks (W_1+ W_2+ $W_3 = W$).

Then

$$W_1 = \eta_1 Q_{A1}, W_2 = \eta_2 Q_{A2}, W_3 = \eta_3 Q_{A3}\ and\ (\eta_1 Q_{A1} + \eta_2 Q_{A2} + \eta_3 Q_{A3}) = W.$$

For Carnot cycle thermal efficiency is done by formula:

$$\eta = 1 - \frac{T_c}{T_h},$$

where T_c - is the temperature of the cold reservoir and T_h - is the absolute temperature of the hot reservoir.

The thermal efficiencies of the cycles:

$$\eta_1 = 1 - \frac{T_4}{T_1}, \eta_2 = 1 - \frac{T_4}{T_2}, \eta_3 = 1 - \frac{T_4}{T_3}.$$

And now

$$\left(\left(1 - \frac{T_4}{T_1}\right) Q_{A1} + \left(1 - \frac{T_4}{T_2}\right) Q_{A2} + \left(1 - \frac{T_4}{T_3}\right) Q_{A3}\right) = W.$$

But we know that

$$W = Q_A - Q_R = Q_{A1} + Q_{A2} + Q_{A3} - Q_R,$$

and $Q_{A1} = 0.6 * Q_{A3}$.

So $W = 0.6 * Q_{A3} + Q_{A2} + Q_{A3} - Q_R = Q_{A2} + 1.6 * Q_{A3} - Q_R$

or $Q_{A2} = W + Q_R - 1.6 * Q_{A3}$.

Substituting Q_{A2} and Q_{A1}in equation for work we get

$$\left(\left(1 - \frac{T_4}{T_1}\right) * 0.6 * Q_{A3} + \left(1 - \frac{T_4}{T_2}\right)(W + Q_R - 1.6 * Q_{A3}) + \left(1 - \frac{T_4}{T_3}\right) Q_{A3}\right) = W.$$

$$Q_{A3} = \frac{W - \left(1 - \frac{T_4}{T_2}\right)(W + Q_R)}{\left(1 - \frac{T_4}{T_1}\right) * 0.6 + \left(1 - \frac{T_4}{T_3}\right) - 1.6 * \left(1 - \frac{T_4}{T_2}\right)}.$$

$$Q_{A3} = \frac{600 - \left(1 - \frac{300}{800}\right) * (600 + 400)}{\left(1 - \frac{300}{1000}\right) * 0.6 + \left(1 - \frac{300}{600}\right) - 1.6 * \left(1 - \frac{300}{800}\right)} = 312.5 \frac{\text{kj}}{min}.$$

$$Q_{A1} = 0.6 * 312.5 = 187.5 \frac{\text{kj}}{min}.$$

$$Q_{A2} = 600 + 400 - 1.6 * 312.5 = 500 \frac{\text{kj}}{min}.$$

Answer: $187.5 \frac{\text{kj}}{min}$, $500 \frac{\text{kj}}{min}$ and $312.5 \frac{\text{kj}}{min}$ respectively.