Objects in the elliptical orbit the position farther from the gravity source is called the apogee and the position nearer is called perigee. What are the speeds of the two objects

Solution

Using the laws of conservation of energy and angular momentum (index ***a*** related to object in apogee, index ***p*** related to object in apogee,)

$$mr_a v_a = mr_p v_p$$

$$v_a = \frac{r_p v_p}{r_a}$$

$$m\frac{{v_a}^2}{2} - G\frac{mM}{r_a} = m\frac{{v_p}^2}{2} - G\frac{mM}{r_p}$$

$$(v_p)^2 - \left(\frac{r_p v_p}{r_a}\right)^2 = 2GM\left(\frac{1}{r_p} - \frac{1}{r_a}\right)$$

$$v_p = \sqrt{GM\left(\frac{1}{r_p} - \frac{1}{r_a}\right)\frac{{r_a}^2}{{r_a}^2 - {r_p}^2}} = \sqrt{GM\frac{r_a}{r_p}\frac{2}{r_a + r_p}}$$

Using the definition of distance between source of gravity and object in apogee and perigee (a is semi-major axis).

$$r_a = a(1+\varepsilon)$$
$$r_p = a(1-\varepsilon)$$
$$\Rightarrow$$

$$v_p = \sqrt{GM\frac{1}{a}\frac{1+\varepsilon}{1-\varepsilon}}$$

$$v_a = \sqrt{GM\frac{1}{a}\frac{1-\varepsilon}{1+\varepsilon}}$$

Answer:

$$v_p = \sqrt{GM\frac{1}{a}\frac{1+\varepsilon}{1-\varepsilon}}$$

$$v_a = \sqrt{GM\frac{1}{a}\frac{1-\varepsilon}{1+\varepsilon}}$$