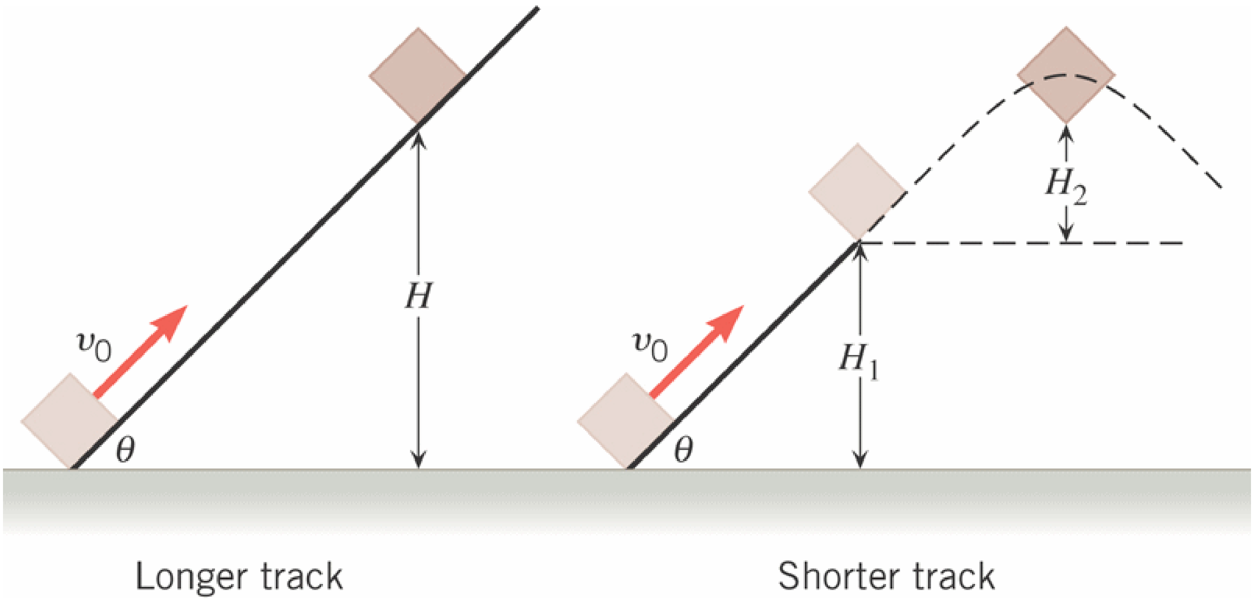
The drawing shows two frictionless inclines that begin at ground level (h = 0 m) and slope upward at the same angle θ. One track is longer than the other, however. Identical blocks are projected up each track with the same initial speed v0. On the longer track the block slides upward until it reaches a maximum height H above the ground. On the shorter track the block slides upward, flies off the end of the track at a height H1 above the ground, and then follows the familiar parabolic trajectory of projectile motion. At the highest point of this trajectory, the block is a height H2 above the end of the track. The initial total mechanical energy of each block is the same and is all kinetic energy. The initial speed of each block is v0 = 7.47 m/s, and each incline slopes upward at an angle of θ = 50.0°. The block on the shorter track leaves the track at a height of H1 = 1.25 m above the ground. Find (a) the height H for the block on the longer track and (b) the total height H1 + H2 for the block on the shorter t

Solution



For the block that stays on the track, its maximal height is attained when all of the kinetic energy is converted to potential energy, or

$$\frac{1}{2}mv_0^2 = mgH \rightarrow H = \frac{v_0^2}{2g} = \frac{\left(7.47\,\frac{\text{m}}{\text{s}}\right)^2}{2 * 9.8\,\frac{\text{m}}{\text{s}^2}} = 2.85\ m.$$

For the other block, at H_1it has lost some E_k:

$$\Delta E_k = \Delta E_p = mgH_1.$$

Then

$$\Delta E_k = \frac{mv_0^2}{2} - \frac{mv^2}{2},$$

so

$$v = \sqrt{v_0^2 - 2gH_1} = \sqrt{\left(7.47\frac{\text{m}}{\text{s}}\right)^2 - 2 * 1.25\text{ m} * 9.8\frac{\text{m}}{\text{s}^2}} = 5.6\frac{m}{s}.$$

We use the projectile motion to find maximal height of the block:

$$H_{max} = H_1 + \frac{v^2\sin^2(\theta)}{2g} = 1.25\text{m} + \frac{\left(5.6\frac{m}{s}\right)^2\sin^2(50º)}{2 * 9.8\frac{\text{m}}{\text{s}^2}} = 2.19\ m.$$

Answer: (a) $\mathbf{2.85}\ \boldsymbol{m}$; (b) $\mathbf{2.19}\ \boldsymbol{m}$.