

1. For the motion of particle, velocity v depends upon displacement x as $v=20/(3x-2)$. If at $t=0$, $x=0$ then at what time t , the $x=20$?

$$v = \frac{20}{3x-2}$$

$$x|_{t=0} = 0$$

$$x_1 = 20$$

$$t_1 = ?$$

Solution.

As it is known, the velocity $v = \frac{dx}{dt}$. Then, we have a differential equation

$$\frac{dx}{dt} = \frac{20}{3x-2}$$

with the initial condition $x|_{t=0} = 0$.

Let's integrate this equation by separating the variables.

$$\frac{dx}{dt} = \frac{20}{3x-2},$$

$$(3x-2)dx = 20dt,$$

$$\int_0^x (3x-2)dx = \int_0^t 20dt,$$

$$\left(\frac{3x^2}{2} - 2x \right) \Big|_0^x = 20t \Big|_0^t,$$

$$\frac{3x^2}{2} - 2x = 20t.$$

So, the time depends upon displacement as

$$t = \frac{1}{20} \left(\frac{3x^2}{2} - 2x \right).$$

Now, we can calculate the time, at which $x = 20$:

$$t_1 = \frac{1}{20} \left(\frac{3x_1^2}{2} - 2x_1 \right).$$

Let evaluate the quantity.

$$t_1 = \frac{1}{20} \left(\frac{3 \cdot 20^2}{2} - 2 \cdot 20 \right) = 28.$$

Answer: the time is equal to 28.