Two identical conducting sphere are fixe in a place attract each other with force of 0.109N separated by 50 cm (center to Center) the sphere are connected by a thin wire. When wire is removed, the sphere attracts each other with a force of 0.0360. What are initial charges on spheres?

Solution

$$F_1 = -\frac{kQq}{r^2} \to Qq = -\frac{F_1 r^2}{k} \to Q = -\frac{1}{q}\frac{F_1 r^2}{k}.$$

But since we are told that the charges attract one another, we know that Q and q have opposite signs and so their product must be negative.

Then the two spheres are joined by a wire, electric potentials of spheres becomes equal. So if the new charge on each sphere is Q_1,

$$\varphi_1 = \varphi_2 = \frac{kQ_1}{R},$$

where R – radius of each sphere. That's why the spheres repel each other with an electrostatic force of 0.0360N, but not attracts.

According to the conservation of charge:

$$Q + q = Q_1 + Q_1 = 2Q_1.$$

$$F_2 = \frac{kQ_1^2}{r^2} \to Q_1 = \sqrt{\frac{F_2 r^2}{k}}.$$

So

$$Q + q = 2Q_1.$$

Now we have

$$q - \frac{1}{q}\frac{F_1 r^2}{k} = 2\sqrt{\frac{F_2 r^2}{k}}.$$

Multiplying this equation by q we get quadratic equation for q:

$$q^2 - 2\sqrt{\frac{F_2 r^2}{k}}\, q - \frac{F_1 r^2}{k} = 0.$$

The discriminant of equation:

$$D = \left(-2\sqrt{\frac{F_2 r^2}{k}}\right)^2 - 4 * 1 * \left(-\frac{F_1 r^2}{k}\right) = 4\frac{F_2 r^2}{k} - 4\left(-\frac{F_1 r^2}{k}\right) = 4\frac{(F_1 + F_2) r^2}{k}.$$

The roots of equation:

$$q_{1,2} = \frac{-\left(-2\sqrt{\frac{F_2 r^2}{k}}\right) \pm \sqrt{4\frac{(F_1+F_2)r^2}{k}}}{2*1} = \frac{2\sqrt{\frac{F_2 r^2}{k}} \pm 2\sqrt{\frac{(F_1+F_2)r^2}{k}}}{2}$$

$$= \sqrt{\frac{r^2}{k}}\left(\sqrt{F_2} \pm \sqrt{(F_1+F_2)}\right) = \sqrt{\frac{0.5^2}{9*10^9}}\left(\sqrt{0.0360} \pm \sqrt{(0.109+0.0360)}\right)$$

$$q_1 = 3*10^{-6}\text{Coulomb}, q_2 = -1*10^{-6}\text{Coulomb}.$$

We have two possible values of charge q. Let's find appropriate values of charge Q:

$$Q_1 = -\frac{1}{q_1}\frac{F_1 r^2}{k} = -\frac{1}{3*10^{-6}} * \frac{0.5^2 * 0.109}{9*10^9} = -1*10^{-6}\text{Coulomb},$$

$$Q_2 = -\frac{1}{q_2}\frac{F_1 r^2}{k} = -\frac{1}{(-1*10^{-6})} * \frac{0.5^2 * 0.109}{9*10^9} = 3*10^{-6}\text{Coulomb}.$$

Answer: $(3*10^{-6}\text{C and } -1*10^{-6}\text{C})$ or $(-1*10^{-6}\text{C and } 3*10^{-6}\text{C})$.