Write the mathematical form of the first law of thermodynamics for (i) a thermally insulated system, and (ii) a perfect gas expanding into vacuum under isothermal conditions.

Answer

W - work done by system.

(i) A thermally insulated system.

The insulation prevents heat from flowing into or out of the system, so any change in the system is adiabatic. The first law of thermodynamics:

$$U_2 - U_1 = \Delta U = -W.$$

When a system expands adiabatically, W is positive (the system doesn't work on its surroundings), ΔU is negative and the internal energy decreases.

(ii) A perfect gas expanding into vacuum under isothermal conditions.

 $U_2 - U_1 = \Delta U = 0 \rightarrow U = const, U(T) = C_v T.$

 C_v - the heat capacity of perfect gas at constant volume.

The gas doesn't work on its surroundings and there is no heat flow through the insulation. So both Q and W are zero and the internal energy is constant.