the centre of the mass of solid cone along the centre of base to vertex is at ?

Solution:

$\mathrm{h}-$ height of the cone;

$\mathrm{m}-$ mass of the cone;

$\mathrm{R}-$ radius of the base of the cone;

First, we can find the density of the cone:

$$\rho = \frac{\mathrm{m}}{\mathrm{V}} = \frac{\mathrm{m}}{\frac{\pi \mathrm{R}^2 \mathrm{h}}{3}} = \frac{3\mathrm{m}}{\pi \mathrm{R}^2 \mathrm{h}}$$

Radius dependence of the height x:

$$\mathrm{r}(\mathrm{x}) = \mathrm{R} \cdot \frac{\mathrm{x}}{\mathrm{h}}, \text{where } \mathrm{x}- \text{ distance to the vertex}$$

Now, let us split the cone on discs height dx. Volume of the disc on the height dx will be:

$$\mathrm{dV} = \pi \mathrm{r}^2 \mathrm{dx} = \frac{\pi \mathrm{R}^2 \mathrm{x}^2 \mathrm{dx}}{\mathrm{h}^2}$$

Mass of this disc will be:

$$\mathrm{dm} = \rho \cdot \mathrm{dV} = \frac{3\mathrm{m}\mathrm{x}^2 \mathrm{dx}}{\mathrm{h}^3}$$

Position of the center of mass is determined by the sum of the $\mathrm{dm} \cdot \mathrm{x}$ divided by the total mass m:

$$\mathrm{x}_{\mathrm{centr}} = \frac{3}{\mathrm{h}^3} \int_0^{\mathrm{h}} \mathrm{x}^3 \mathrm{dx} = 3 \cdot \frac{\mathrm{h}}{4}.$$

Hence, the center of mass is located at a distance $\frac{3}{4}$ the height of the cone of the vertex or $\frac{\mathrm{h}}{4}$ of the center of base.

Answer: center of mass of the solid cone is located at a distance $\frac{3}{4}$ the height of the cone of the vertex or $\frac{1}{4}h$ of the center of base.