a disc and a ring of same mass are rolling down fritionless inclined plane with same kinetic energy .the ratio of velocity of disc to velocity of ring is ?

Solution:

Kinetic energies of the rind and disc are equal:

$$E_{disc} = E_{ring} \qquad (1)$$

The kinetic energy of rotational motion:

$$E = \frac{J \cdot \omega^2}{2} + \frac{m \cdot \vartheta^2}{2}$$

, where $J -$ moment of inertia, $\omega = \frac{\vartheta}{r} -$ angular velocity, $r -$ radius, $m -$ mass.

Moment of inertia of the disc and the ring:

$$J_{disk} = \frac{mr_{disc}^2}{2} \Longrightarrow$$

$$E_{disc} = \frac{mr_{disc}^2}{2 \cdot 2} \cdot \omega_{disc}^2 + \frac{m \cdot \vartheta_{disc}^2}{2} = \frac{mr_{disc}^2}{4}\left(\frac{\vartheta_{disc}}{r_{disc}}\right)^2 + \frac{m \cdot \vartheta_{disc}^2}{2} = \frac{m\vartheta_{disc}^2}{4} + \frac{m \cdot \vartheta_{disc}^2}{2}$$

$$= \frac{3m \cdot \vartheta_{disc}^2}{2} \qquad (2)$$

$$J_{ring} = mr^2 \Longrightarrow$$

$$E_{ring} = \frac{mr_{ring}^2}{2} \cdot \omega_{ring}^2 + \frac{m \cdot \vartheta_{ring}^2}{2} = \frac{mr_{ring}^2}{4}\left(\frac{\vartheta_{ring}}{r_{ring}}\right)^2 + \frac{m \cdot \vartheta_{ring}^2}{2} =$$

$$= \frac{m\vartheta_{ring}^2}{2} + \frac{m \cdot \vartheta_{ring}^2}{2} = m \cdot \vartheta_{ring}^2 \qquad (3);$$

(3)and(2)in(1):

$$\frac{3m \cdot \vartheta_{disc}^2}{2} = m \cdot \vartheta_{ring}^2$$

$$\frac{\vartheta_{disc}^2}{\vartheta_{ring}^2} = \frac{2}{3}$$

$$\frac{\vartheta_{disc}}{\vartheta_{ring}} = \sqrt{\frac{2}{3}} = 0.8$$

Answer: the ratio of velocity of disc to velocity of ring is 0.8.