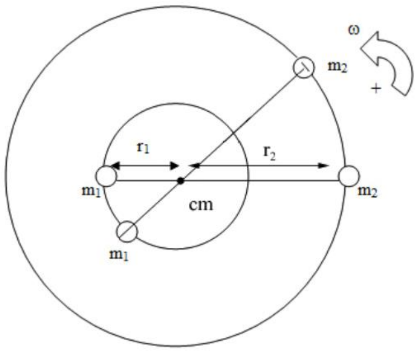
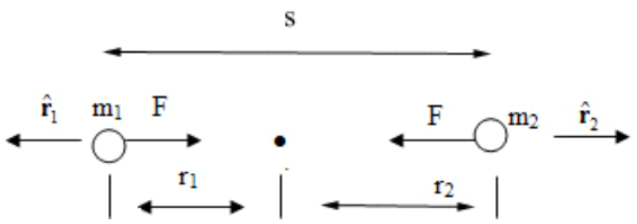
Consider a double star system under the influence of the gravitational force between the stars. Star 1 has mass m1 = 1.42 × 1031 kg and Star 2 has mass m2 = 2.62 × 1031 kg. Assume that each star undergoes uniform circular motion about the center of mass of the system (cm). In the figure below r1 is the distance between Star 1 and cm, and r2 is the distance between Star 2 and cm.If the stars are always a fixed distance s=r1+r2 = 3.48 × 1018 m apart, what is the period of the orbit (in s)?

Solution



Choose radial coordinates for each star with origin at center of mass. Let $\hat{r}_1$be a unit vector at Star 1 pointing radially away from the center of mass. Let $\hat{r}_2$beaunit vector at Star 2 pointing radially away from the center of mass. The force diagrams on
the two stars are shown in the figure below.



From Newton's Second Law, $\overline{F_1} = m_1\overline{a_1}$, for Star 1 in the radial direction is

$$\hat{r}_1: -G\frac{m_1 m_2}{s^2} = -m_1 r_1 \omega^2.$$

We can solve this for r_1,

$$r_1 = G\frac{m_2}{\omega^2 s^2}.$$

Newton's Second Law, $\overline{F_2} = m_2\overline{a_2}$, for Star 2 in the radial direction is

$$\hat{r}_2: -G\frac{m_1 m_2}{s^2} = -m_1 r_1 \omega^2.$$

We can solve this for r_2,

$$r_2 = G\frac{m_1}{\omega^2 s^2}.$$

Since s, the distance between the stars, is constant

$$s = r_1 + r_2 = G\frac{m_1}{\omega^2 s^2} + G\frac{m_1}{\omega^2 s^2} = G\frac{(m_2 + m_1)}{\omega^2 s^2}.$$

Thus the angular velocity is

$$\omega = \left(G\frac{(m_2 + m_1)}{s^3}\right)^{\frac{1}{2}}$$

and the period is then

$$T = \frac{2\pi}{\omega} = \left(\frac{4\pi^2 s^3}{G(m_2 + m_1)}\right)^{\frac{1}{2}}.$$

$$T = \sqrt{\frac{4\pi^2\left(3.48 \times 10^{18}\right)^3}{6.67 \times 10^{-11}\left(1.42 \times 10^{31} + 2.62 \times 10^{31}\right)}} = 7.85 * 10^{17} s.$$

Answer: $\mathbf{7.85 * 10^{17} s}$.