Question 36330

Let us first write the first law of thermodynamics $\delta Q = dU + p\,dV$.
For constant pressure, $\delta Q = dU$. Hence, heat capacity for constant pressure is

$$C_V = \left(\frac{\delta Q}{dT}\right)_V = \left(\frac{\partial U}{\partial T}\right)_V .$$

For an ideal gas, internal energy U is the function only of temperature ($U = U(T)$). Hence,

$$C_V = \frac{dU}{dT} .$$

Now, by definition

$$C_P = \left(\frac{\delta Q}{dT}\right)_P = \left(\frac{dU + p\,dV}{dT}\right)_P = \left(\frac{\partial U}{\partial T}\right)_P + P\left(\frac{\partial V}{\partial T}\right)_P .$$

For one mole of ideal gas, equation of state is $PV = RT$, which yields $\left(\frac{\partial V}{\partial T}\right)_P = \frac{R}{P}$ and $\left(\frac{\partial U}{\partial T}\right)_P = \frac{dU}{dT} = C_V$. Plugging in last two equalities into formula for C_P , obtain

$C_P = C_V + R$ - this is Mayer's formula.

This formula works only for ideal gas, because we have used the equation $PV = RT$ for ideal gas, while deriving it.