A satellite has a mass of 6403 kg and is in a circular orbit 4.42 × 105 m above the surface of a planet. The period of the orbit is 2.3 hours. The radius of the planet is 4.73 × 106 m. What would be the true weight of the satellite if it were at rest on the planet's surface?

Solution

The height of orbit H. The radius of the planet - R. Distance between the planet and the satellite $H + R$.

The centripetal force F_c is provided by the force of gravityF_g :

$$F_g \ = \ F_c \rightarrow \frac{GMm}{(H+R)^2} = m\omega^2(H+R),$$

where G – constant of gravity, M - the mass of the planet, m - mass of satellite, ω – angular velocity of the orbit.

We know that $\omega = \frac{2\pi}{T}$, where T - the period of the orbit.

Now we have

$$\frac{GMm}{(H+R)^2} = \frac{4\pi^2 m(H+R)}{T^2} \rightarrow \frac{GM}{(H+R)^2} = \frac{4\pi^2(H+R)}{T^2}.$$

The mass of the planet

$$M \ = \frac{4\pi^2((H+R)^3)}{T^2 G} = \frac{4\pi^2(4.42 * 10^5 + 4.73 * 10^6)^3}{\left(2.3\ \mathrm{h} * 3600\,\frac{\mathrm{s}}{\mathrm{h}}\right)^2 * 6.67 * 10^{-11}} = 1.19 * 10^{24} kg.$$

The gravitational force, F_g, is also equal to $m * g$. To find the acceleration,

$$mg = \frac{GMm}{R^2} \rightarrow g = \frac{GM}{R^2}.$$

At the surface of the planet,

$$g = \frac{6.67 * 10^{-11} * 1.19 * 10^{24}}{(4.73 * 10^6)^2} = 3.55\,\frac{\mathrm{m}}{\mathrm{s}^2}.$$

So,

$$F_g = mg = 6403 * 3.55 = 23\ \mathrm{kN}.$$

Answer: 23 kN.