1. A stone is tied to a weightless string and revolved in a vertical circle of radius 5m. (i) What should be the minimum speed of the stone at the highest point of the circle so that the string does not slack? (ii) What should be the speed of the stone at the lowest point in this situation?

$r = 5\,m$ $g = 9.8\frac{m}{s^2}$	*Solution.* The string begins to slack, if its weight force is equal to zero, so the acceleration of the stone (the centripetal acceleration during the movement around the circle) straights to the acceleration of the free fall due to the gravity. $m \cdot \frac{v_{\min}}{r^2} = mg$.
$v_{\min}, v_1 - ?$	

One can find this minimal speed: $\boxed{v_{\min} = \sqrt{g\,r}}$.

In this situation, we can express the speed of the stone at the lowest point, using the law of conservation and transformation of energy: the increasing of the kinetic energy is due to the decreasing he potential energy of the stone.

$$\frac{m v_1^{\,2}}{2} - \frac{m v_{\min}^{\,2}}{2} = mg \cdot 2r\,, \quad v_1^{\,2} = 4\,g\,r + v_{\min}^{\,2}\,, \quad v_1 = \sqrt{4\,g\,r + v_{\min}^{\,2}} = \sqrt{4\,g\,r + g\,r}\,, \quad \boxed{v_1 = \sqrt{5\,g\,r}}.$$

Let check the dimension.

$$[v_{\min}] = [v_1] = \sqrt{\frac{m}{s^2} \cdot m} = \sqrt{\frac{m^2}{s^2}} = \frac{m}{s}\,.$$

Let evaluate the quantities.

$$v_{\min} = \sqrt{9.8 \cdot 5} = 7\left(\frac{m}{s}\right), \quad v_1 = \sqrt{5 \cdot 9.8 \cdot 5} \approx 15.7\left(\frac{m}{s}\right).$$

Answer: $7\frac{m}{s}$, $15.7\frac{m}{s}$.