Is average speed the magnitude of average velocity? Justify your answer with a suitable example.

Answer

In kinematics, the speed of an object is the magnitude of its velocity. But an average speed is not the magnitude of average velocity.

Average speed defined by the formula:

$$\bar{v} = \frac{d}{t},$$

where d is total distance travelled by the object for the time t.

Average velocity is described by the formula:

$$\bar{\vec{v}} = \frac{\vec{d}}{t},$$

where $\vec{d}$ is the displacement of the object for the time t.

Average velocity magnitudes are always smaller than or equal to average speed of a given object.

Example:

Let us consider a car travelling to the north 30 km 4 hours and 40 km to the west 3 hours.

So, the average speed is

$$\bar{v} = \frac{30\ \text{km} + 40\ \text{km}}{4\ \text{h} + 3\ \text{h}} = \frac{70\ \text{km}}{7\ \text{h}} = 10\frac{km}{h}.$$

The average velocity's magnitude is

$$\left|\bar{\vec{v}}\right| = \frac{\sqrt{(30\ \text{km})^2 + (40\ \text{km})^2}}{4\ \text{h} + 3\ \text{h}} = \frac{50\ \text{km}}{7\ \text{h}} = 7.1\frac{km}{h}.$$

As we see average velocity's magnitude is smaller than average speed.