We are standing at a distance d=15 m away from a house. The house wall is h=6 m high and the roof has an inclination angle β=30 ∘ . We throw a stone with initial speed v0=20 m/s at an angle α= 38 ∘ . The gravitational acceleration is g=10 m/s2.
(a) At what horizontal distance from the house wall is the stone going to hit the roof - s in the figure-? (in meters)
(b) What time does it take the stone to reach the roof? (in seconds)

Solution:

The equation of motion for the stone for the X-axis:

$$x: s + d = V_0 t \cos\alpha$$

$$t = \frac{s + d}{V_0 \cos\alpha} \qquad (1)$$

The equation of motion for the stone for the Y-axis:

$$y: h + h_1 = V_0 t \sin\alpha - \frac{gt^2}{2} \qquad (2)$$

From the right triangle ABC:

$$\tan\beta = \frac{h_1}{s} \Longrightarrow h_1 = s\tan\beta \qquad (3)$$

$(3) \text{in} (2)$:

$$h + s\tan\beta = V_0 t \sin\alpha - \frac{gt^2}{2} \qquad (4)$$

$(1) \text{in} (4)$:

$$h + s\tan\beta = \left(\frac{V_0 \sin\alpha}{V_0 \cos\alpha}\right)(s + d) - \frac{g}{2} \cdot \left(\frac{s + d}{V_0 \cos\alpha}\right)^2$$

$$h + s\tan\beta = (s + d)\tan\alpha - \frac{g}{2} \cdot \left(\frac{s + d}{V_0 \cos\alpha}\right)^2$$

$$h + s\tan\beta = s\tan\alpha + d\tan\alpha - \frac{gs^2}{2(V_0 \cos\alpha)^2} - \frac{sdg}{(V_0 \cos\alpha)^2} - \frac{gd^2}{2(V_0 \cos\alpha)^2}$$

$$s^2 \cdot \frac{g}{2(V_0 \cos\alpha)^2} + s\left(\tan\beta - \tan\alpha + \frac{dg}{(V_0 \cos\alpha)^2}\right) + h - d\tan\alpha + \frac{gd^2}{2(V_0 \cos\alpha)^2} = 0$$

$$0.02013s^2 + 0.4s - 3.455 = 0$$

$$s = 6.5\text{m};\ -26.38\text{m}$$

We need only the positive root of the equation:

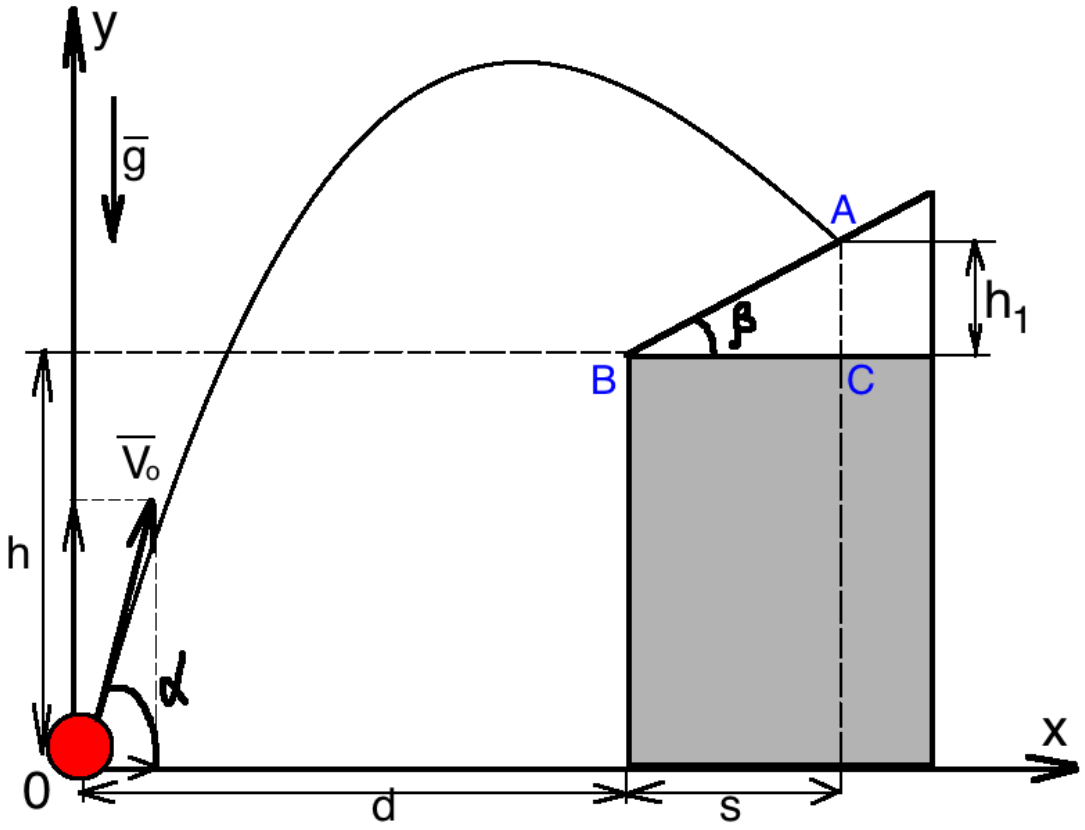
$$s = 6.5\text{m}$$

Time to reach the roof:

$$(1): t = \frac{6.5\text{m} + 15\text{m}}{20\frac{\text{m}}{\text{s}} \cdot \cos 38^\circ} = 1.36\text{s}$$

Answer: a) the stone is going to hit the roof at distance $s = 6.5\text{m}$ from the house wall;

b)time to reach the roof $t = 1.36s$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