The maximum vertical distance through which a fully dressed astronaut can jump on the earth is 0.5 m. If mean density of the moon is two thirds that of the earth and radius is one quarter that of the earth, the maximum vertical distance through which he can jump on the moon and the ratio of time of duration of the jump on the moon to that on the earth are :

Solution:

$G = 6.67 \cdot 10^{-11} \frac{N \cdot m^2}{kg^2}$ – gravitational constant;

ρ_E – density of the earth;

$\rho_M = \frac{2}{3}\rho_E$ – density of the moon;

R_E – radius of the earth;

$R_M = \frac{1}{4}R_E$ – radius of the moon;

$g_E = 9.8\frac{m}{s^2}$ – gravitational acceleration on the earth;

g_M – gravitational acceleration on the moon;

$h_E = 0.5m$ – maximum vertical distance of earth;

h_M – maximum vertical distance of moon;

First, we need to find gravitational acceleration on the moon:

$$g_E = \frac{GM_E}{R_E^2} = \frac{G \cdot \rho_E \cdot \frac{4}{3}\pi R_E^3}{R_E^2} = G \cdot \rho_E \cdot \frac{4}{3}\pi R_E \quad (1)$$

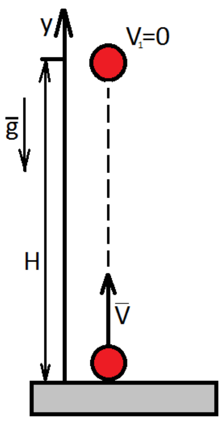
$$g_M = \frac{GM_M}{R_E^2} = \frac{G \cdot \rho_M \cdot \frac{4}{3}\pi R_M^3}{R_M^2} = G \cdot \rho_M \cdot \frac{4}{3}\pi R_M = G \cdot \frac{2}{3}\rho_E \cdot \frac{4}{3}\pi \cdot \frac{1}{4}R_E \quad (2)$$

$(2) \div (1)$:

$$\frac{g_M}{g_E} = \frac{G \cdot \frac{2}{3}\rho_E \cdot \frac{4}{3}\pi \cdot \frac{1}{4}R_E}{G \cdot \rho_E \cdot \frac{4}{3}\pi R_E} = \frac{2 \cdot 1}{3 \cdot 4} = \frac{1}{6}$$

$$g_M = \frac{g_E}{6} \quad (3)$$

Now we need to find the speed with which the astronaut jumped on earth:



The equation of motion for the astronaut:

$$h_E = Vt_E - \frac{g_E t_E{}^2}{2} \; (1)',$$

t_E – flight time to reach the height $h_E = 0.5m$

Rate equation for the astronaut, in the end of the flight speed of the astronaut is zero:

$$0 = V - g_E t_E$$

$$t_E = \frac{V}{g_E} \; (2)'$$

$$(2)' \text{in } (1)': h_E = V\frac{V}{g_E} - \frac{g_E\left(\frac{V}{g_E}\right)^2}{2}$$

$$h_E = \frac{V^2}{2g_E}$$

$$V = \sqrt{2g_E h_E}$$

Residence time in the air: time of flight of the ball upward equals time of the fall:

$$T_E = t_{up} + t_{down} = 2t = \frac{2V}{g_E}$$

The maximum vertical distance through which he can jump on the moon is:

$$h_M = \frac{V^2}{2g_M} = 6h_E = 3m$$

Ratio of time of duration of the jump on the moon to that on the earth:

$$\frac{T_M}{T_E} = \frac{2V}{g_M} \cdot \frac{g_E}{2V} = \frac{g_E}{\frac{g_E}{6}} = 6$$

Answer: The maximum vertical distance through which he can jump on the moon is 3m. Ratio of time of duration of the jump on the moon to that on the earth is 6.