Task. A truck covers 40.0 m in 8.5 s while smoothly accelerating to a final velocity of 2.8 m/s. What was the original velocity, v_0? What is the acceleration, a?

Solution. Assume that the acceleration of the truck is constant. Then the distance $d(t)$ covered by truck at time t, and the velocity $v(t)$ at the same time are given by the following formulas:

$$d(t) = v_0 t + \frac{at^2}{2},$$
$$v(t) = v_0 + at.$$

By assumption,

$$d(8.5\ s) = 40.0\ m,$$

and

$$v(8.5\ s) = 2.8\ m/s.$$

Denote

$$t = 8.5\ s,$$
$$d_1 = d(t_1) = d(8.5\ s) = 40.0\ m, \qquad v_1 = v(t_1) = v(8.5\ s) = 2.8\ m/s.$$

Then we obtain the following system of equations:

$$v_0 t + \frac{at^2}{2} = d_1, \qquad v_0 + at = v_1.$$

From the second equation we get:

$$at = v_1 - v_0.$$

Substituting this formula into the first equation we obtain

$$v_0 t + \frac{(v_1 - v_0)t}{2} = d_1,$$

whence

$$\frac{2v_0 + v_1 - v_0}{2} \cdot t = d_1,$$
$$\frac{v_0 + v_1}{2} = \frac{d_1}{t},$$
$$v_0 = \frac{2d_1}{t} - v_1,$$

and therefore

$$a = \frac{v_1 - v_0}{t} = \frac{v_1 - \frac{2d_1}{t} + v_1}{t} = \frac{2v_1 t - 2d_1}{t^2}$$
$$= \frac{2(v_1 t - d_1)}{t^2}.$$

Substituting values we get:

$$v_0 = \frac{2d_1}{t} - v_1 = \frac{2 \cdot 40.0}{8.5} - 2.8 \approx 6.1\ m/s,$$
$$a = \frac{2(v_1 - d_1 t)}{t^2} = \frac{2 \cdot (2.8 \cdot 8.5 - 40.0)}{8.5^2} \approx -0.45\ m/s^2.$$

Answer. $v_0 = 6.1\ m/s, \quad a = -0.45\ m/s^2.$