Write the equation of motion of a simple harmonic oscillator which has an amplitude of
5 cm and it executes 150 oscillations in 5 minutes with an initial phase of 45°. Also obtain the value of its maximum velocity.

Solution:

$A = 5\text{ cm} = 0.05\text{ m} - \text{amplitude of harmonic oscillations;}$

$\varphi_0 = 45° = \frac{\pi}{4} - \text{initial phase ;}$

Equation of motion of a simple harmonic oscillator:

$$x(t) = A\cos(\omega t + \varphi_0)$$

To find the equation to find the cyclic frequency. Cyclic frequency shows how many radians moved the body per unit of time:

$$\omega = 2\pi \cdot f = 2\pi \cdot \frac{150\text{ oscillations}}{5 \cdot 60\text{s}} = \pi\text{ Hz} \approx 3.14\text{ Hz} \Rightarrow$$

$$x(t) = A\cos(\omega t + \varphi_0) = 0.05\left(\pi t + \frac{\pi}{4}\right).$$

To find the maximum speed, we need to take the derivative of $x(t)$:

$$V(t) = x'(t)$$

$$V(t) = -A\omega\sin(\omega t + \varphi_0)$$

$-1 \le \sin(\omega t + \varphi_0) \le 1 \Rightarrow$ maximum speed when the sine is equal to 1 or -1:

$$V_{max} = -A\omega \cdot (-1) = A\omega = 0.05\text{m} \cdot \pi = 0.157\frac{\text{m}}{\text{s}}$$

Answer: Equation of motion of a simple harmonic oscillator:

$x(t) = 0.05\left(\pi t + \frac{\pi}{4}\right)$. maximum speed: $V_{max} = 0.157\frac{\text{m}}{\text{s}}$.