The gravitational force that the sun exerts on the moon is perpendicular to the force that the earth exerts on the moon. The masses are: mass of sun=1.99 × 1030 kg, mass of earth=5.98 × 1024 kg, mass of moon=7.35 × 1022 kg. The distances shown in the drawing are rSM = 1.50 × 1011 m and rEM = 3.85 × 108 m. Determine the magnitude of the net gravitational force on the moon.

Solution:

$\mathrm{M_s} = 1.99 \times 10^{30}\mathrm{kg} -$ mass of Sun;

$\mathrm{M_e} = 5.98 \times 10^{24}\mathrm{kg} -$ mass of Earth;

$\mathrm{M_m} = 7.35 \times 10^{22}\mathrm{kg} -$ mass of Moon;

$\mathrm{r_{SM}} = 1.50 \times 10^{11}\mathrm{m} -$ distance to the Moon from the Sun;

$\mathrm{r_{EM}} = 3.85 \times 10^{8}\mathrm{m} -$ distance to the Moon from the Earth;

$\mathrm{G} = 6.67 \times 10^{-11}\mathrm{N}\left(\frac{\mathrm{m}}{\mathrm{kg}}\right)^2 -$ gravitational constant

The gravitational force that acts on the Moon by the Earth (Law of Gravity):

$$\mathrm{F_e} = \mathrm{G}\frac{\mathrm{M_e} \cdot \mathrm{M_m}}{\mathrm{r_{EM}^2}} = 6.67 \times 10^{-11}\mathrm{N}\left(\frac{\mathrm{m}}{\mathrm{kg}}\right)^2 \cdot \frac{5.98 \times 10^{24}\mathrm{kg} \cdot 7.35 \times 10^{22}\mathrm{kg}}{\left(3.85 \times 10^{8}\mathrm{m}\right)^2} = 1.98 \times 10^{20}\mathrm{N}$$

The gravitational force that acts on the Moon by the Sun (Law of Gravity):

$$\mathrm{F_S} = \mathrm{G}\frac{\mathrm{M_s} \cdot \mathrm{M_m}}{\mathrm{r_{SM}^2}} = 6.67 \times 10^{-11}\mathrm{N}\left(\frac{\mathrm{m}}{\mathrm{kg}}\right)^2 \cdot \frac{1.99 \times 10^{30}\mathrm{kg} \cdot 7.35 \times 10^{22}\mathrm{kg}}{\left(1.50 \times 10^{11}\mathrm{m}\right)^2} = 4.34 \times 10^{20}\mathrm{N}$$

Net gravitational force on the moon:

$$\vec{\mathrm{F}} = \overrightarrow{\mathrm{F_e}} + \overrightarrow{\mathrm{F_S}}$$

Pythagorean theorem for a right triangle ABC:

$$\mathrm{F} = \sqrt{\mathrm{F_S^2} + \mathrm{F_e^2}} = \sqrt{(1.98 \times 10^{20}\mathrm{N})^2 + (4.34 \times 10^{20}\mathrm{N})^2} = 4.77 \times 10^{20}\mathrm{N}$$

Answer: magnitude of the net gravitational force on the moon is $4.77 \times 10^{20}\mathrm{N}$.