

Relative to the ground, a car has a velocity of 16.7 m/s, directed due north. Relative to this car, a truck has a velocity of 25.0 m/s, directed 52.0° north of east. What is the magnitude of the truck's velocity relative to the ground?

Solution:

V_t – velocity of the truck relative to the ground;

$V_{t,c} = 25.0 \frac{m}{s}$ – velocity of the truck relative to the car;

$V_c = 16.7 \frac{m}{s}$ – velocity of the car relative to the ground;

$\alpha = 52.0^\circ$ – the angle between the west
– east direction and the direction of the truck

Formula for the relative velocity of the truck:

$$\vec{V}_{t,c} = \vec{V}_t - \vec{V}_c$$

$$\vec{V}_t = \vec{V}_{t,c} + \vec{V}_c$$

The cosine theorem in the triangle ABC ($\beta = 180^\circ - (90^\circ - \alpha) = 90^\circ + \alpha$):

$$V_t = \sqrt{V_{t,c}^2 + V_c^2 - 2V_{t,c} \cdot V_c \cdot \cos \beta} = \sqrt{V_{t,c}^2 + V_c^2 - 2V_{t,c} \cdot V_c \cdot \cos(90^\circ + \alpha)};$$

$$\cos(90^\circ + \alpha) = -\sin \alpha \Rightarrow$$

$$V_t = \sqrt{\left(16.6 \frac{m}{s}\right)^2 + \left(25 \frac{m}{s}\right)^2 + 2 \left(16.6 \frac{m}{s}\right) \cdot \left(25 \frac{m}{s}\right) \cdot \sin 52^\circ} = 39.43 \frac{m}{s}$$

Answer: magnitude of the truck's velocity relative to the ground is $39.43 \frac{m}{s}$

