if a disc of radius 5m is such that its density varies from its centre proportional to the sqr of distance ..
density at distance 1 is 2kg/msqr then determine its MI about an axis through centre and perpendicular the plane

Solution:

Density at distance 1m:

$\rho_1 = \alpha {r_1}^2 = 2\frac{kg}{m^2} = \alpha(1m)^2 \Rightarrow \alpha = 2\frac{kg}{m^4}$

$\rho = \alpha r^2$ – density of the disc;

$R = 5m$ - radius of the disk.

Let's take any element of the disc, which is located at a distance r (see figure) and find the mass this element. For the point located at a distance r, $\rho = \alpha r^2$. Mass of an element is equal to the product of area and area density:

$$dL = r\,d\varphi;$$

$$dm = \alpha r^2 \cdot dS = \alpha r^2 \cdot dr\,dL = \alpha r^2 \cdot r\,dr\,d\varphi = \alpha r^3\,dr\,d\varphi$$

Moment of inertia of the element relative to the selected axis through centre and perpendicular to the plane:

$$dJ = dm\,r^2 = \alpha r^3 dr\,d\varphi\,r^2 = \alpha r^5\,dr\,d\varphi$$

To find the moment of inertia of the disk, we need to sum the moment of inertia of the element over the entire disk, so we need to take the integral of the ring, which elementary part is our element dJ ($0 \le r \le R, 0 \le \varphi \le 2\pi$):

$$J = \int_0^{2\pi}\int_0^R dJ\,;$$

$$J = \int_0^{2\pi}\int_0^R \alpha r^5\,dr\,d\varphi = \alpha\int_0^{2\pi} d\varphi \int_0^R r^5 dr = \alpha\pi r^6|_0^R = \alpha\pi R^6$$

$$= 3.14 \cdot 2\,\frac{kg}{m^4} \cdot (5m)^6 = 9.8 \times 10^4 kg \cdot m^2$$

Answer: Moment of inertia of the disc is $9.8 \times 10^4 kg \cdot m^2$.