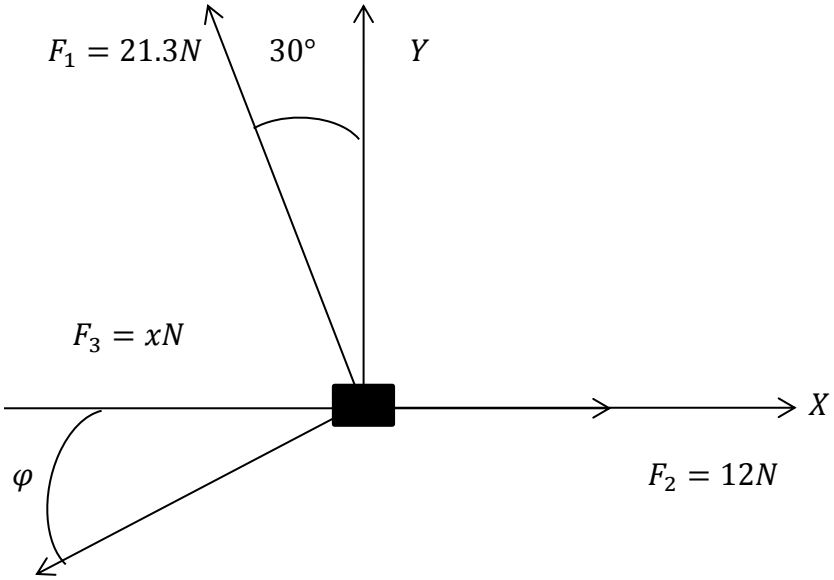

Solve. Three forces are applied to an object, as shown in the figure. Force has a magnitude of 21.3 newtons (N) and is directed 30.0° to the left of the +y axis. Force has a magnitude of 12.0 N and points along the +x axis. A third force is applied such that the vector sum of the three forces is 0 N. What are (a) the magnitude and (b) direction of this third force? Specify the direction as a positive angle relative to the negative x axis, as shown in the figure.

Solution.

A third force is applied such that the vector sum of the three forces is 0 N. So we can state:

$$\overrightarrow{F_1} + \overrightarrow{F_2} + \overrightarrow{F_3} = 0$$

First, project onto the X axis:

$$-F_1 \cdot \sin 30° + F_2 - F_3 \cdot \cos \varphi = 0$$

Then project onto the$Y$ axis:

$$F_1 \cdot \cos 30° - F_3 \cdot \sin \varphi = 0$$

So we have the system of two equations with two unknown:

$$\begin{cases} -F_1 \cdot \sin 30° + F_2 - F_3 \cdot \cos \varphi = 0 \\ F_1 \cdot \cos 30° - F_3 \cdot \sin \varphi = 0 \end{cases}$$

Rewrite:

$$\begin{cases} F_3 \cdot \cos \varphi = -F_1 \cdot \sin 30° + F_2 \\ F_3 \cdot \sin \varphi = F_1 \cdot \cos 30° \end{cases}$$

Divide second equation by first:

$$\begin{cases} \tan\varphi = \dfrac{F_1\cos 30^\circ}{F_2 - F_1\sin 30^\circ} \\ F_3 = \dfrac{F_1\cos 30^\circ}{\sin\varphi} \end{cases}$$

$$\begin{cases} \tan\varphi = \dfrac{21.3\cdot\frac{\sqrt{3}}{2}}{-21.3\cdot\frac{1}{2}+12} = 13.1 \\ F_3 = \dfrac{21.3\cdot\frac{\sqrt{3}}{2}}{\sin(\arctan 13.1)} \approx 18.4 \end{cases}$$

$$\begin{cases} \varphi = \arctan 0.8145 = 85.6^\circ \\ F_3 = \dfrac{18.45}{\sin(85.6^\circ)} \end{cases}$$

Answer:

the magnitude of the third force is

$$F_3 = 26.4\ N$$

positive angle relative to the negative X axis is

$$\varphi = 85.6^\circ$$