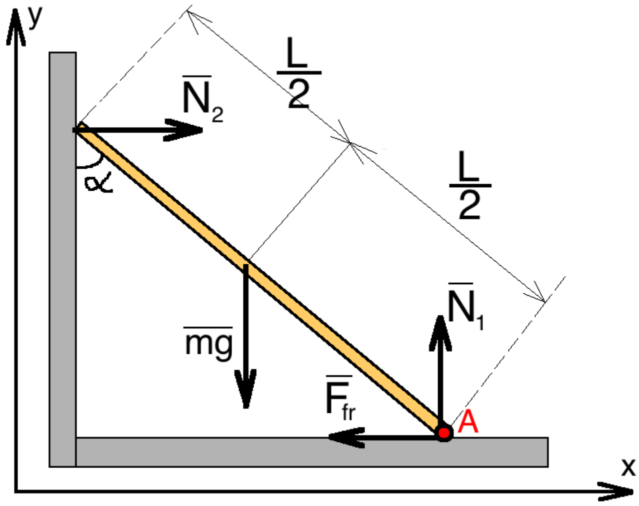
A uniform ladder of mass 10 kg leans against a smooth vertical wall making an angle of 53 with it.The other end rests on a rough horizontal floor,Find the normal force and the frictional force that the floor exerts on the ladder.

Solution:



F_{fr} – frictional force that the floor exerts on the ladder;

$\alpha = 53^o$ – angle which ladder makes with the wall

N_1 – reaction force from the floor

N_2 – reaction force from the wall

$m = 10kg$ – mass of the ladder

L – length of the ladder

Newton's second law for the ladder (the first law of equilibrium):

$\overrightarrow{F_{fr}} + m\vec{g} + \overrightarrow{N_1} + \overrightarrow{N_2} = \vec{0}$

Projection of the law on the X-axis:

$x: N_2 - F_{fr} = 0$ (1)

Projection of the law on the Y-axis:

$y: N_1 - mg = 0$ (2)

Normal force that the floor exerts on the ladder:

$$N_1 = mg = 10kg \cdot 9.8\frac{m}{s^2} = 98N$$

Momentum equation for point A (the second law of equilibrium):

$A: M_{mg} + M_{fr} + M_{N_2} = 0$ (3)

$(M_{N_1} = M_{fr} = 0$, because moment arm of this forces is zero)

$M_{mg} = -mg \cdot \frac{L}{2}\sin\alpha$ (minus sign because of the direction of force)

$M_{N_2} = N_2 \cdot L\cos\alpha$

$\rightarrow (3)$:

$$N_2 \cdot L\cos\alpha - mg \cdot \frac{L}{2}\sin\alpha = 0$$

$$N_2 = \frac{mg}{2} \cdot \frac{\sin\alpha}{\cos\alpha} = \frac{mg\tan\alpha}{2}$$

$\rightarrow (1)$

$$F_{fr} = N_2 = \frac{mg\tan\alpha}{2} = \frac{10\ \mathrm{kg} \cdot 9.8\ \frac{\mathrm{N}}{\mathrm{kg}} \cdot \tan 53^{o}}{2} = 65\mathrm{N}$$

Answer: Normal force that the floor exerts on the ladder: $N_1 = 98\mathrm{N}$;

frictional force that the floor exerts on the ladder $F_{fr} = 65\mathrm{N}$