Task. While driving home from school you travel at $v_1 = 95$ km/h for $d_1 = 130$ km then slow to $v_2 = 65$ km/h. You get home in $t = 3$ hours and 20 min. How far is your hometown from school and what is the average speed?

Solution. Let t_1 be the time during which I travelled with velocity v_1 for the distance d_1, t_2 be the time when I travelled the rest of the distance with velocity v_2. Then

$$t = t_1 + t_2, \qquad t_1 = \frac{d_1}{v_1}.$$

The distance which I travelled with velocity v_2 is equal to

$$d_2 = v_2 t_2.$$

We should find

$$d = d_1 + d_2,$$

and the average velocity v

$$v = \frac{d_1 + d_2}{t_1 + t_2} = \frac{d}{t}.$$

From the above formulas it follows that

$$t_2 = t - t_1 = t - \frac{d_1}{v_1}$$

$$d_2 = v_2 t_2 = v_2 \left(t - \frac{d_1}{v_1} \right) = t v_2 - d_1 v_2 / v_1.$$

Hence

$$d = d_1 + d_2 = d_1 + t v_2 - d_1 v_2 / v_1 = t v_2 + d_1 (1 - v_2 / v_1) = t v_2 + \frac{d_1 (v_1 - v_2)}{v_1}$$

$$v = \frac{d}{t} = \frac{t v_2 + \frac{d_1 (v_1 - v_2)}{v_1}}{t} = v_2 + \frac{d_1 (v_1 - v_2)}{t v_1}.$$

Substituting values we get

$$t = 3 \text{ hours and } 20 \text{ min} = 3\frac{1}{3} = \frac{10}{3} \text{ hours}$$

$$d = t v_2 - \frac{d_1 (v_1 - v_2)}{v_1} = \frac{10}{3} \cdot 65 + \frac{130 \cdot (95 - 65)}{95} = \frac{650}{3} + \frac{3900}{95} =$$

$$= 216\frac{2}{3} + 41\frac{1}{19} = 257\frac{38 + 3}{3 \cdot 19} = 257\frac{41}{57} \approx 260 \text{ } km,$$

$$v = \frac{d}{t} = \frac{257\frac{41}{57}}{10/3} = \frac{14690/57}{10/3} = \frac{14690 \cdot 3}{57 \cdot 10} = \frac{1469}{19} = 77\frac{6}{19} \approx 77 \text{ } km/hour.$$

Answer. $d \approx 260$ km, $v = 77$ $km/hour$.