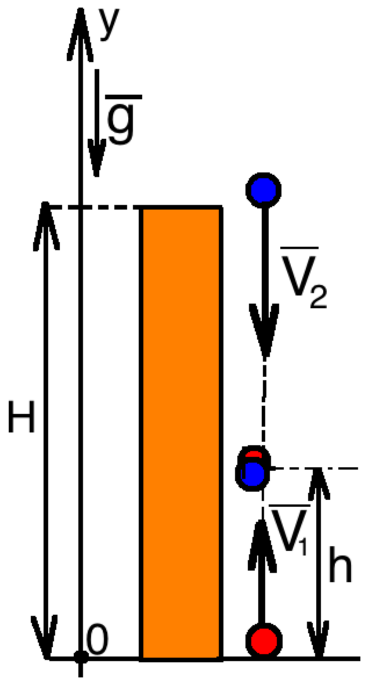
Two stones are thrown simultaneously, one straight upward from the base of a cliff and the other straight downward from the top of the cliff. The height of the cliff is 6.63 m. The stones are thrown with the same speed of 8.97 m/s. Find the location (above the base of the cliff) of the point where the stones cross paths.

Solution:



$V_1 = 8.97\frac{m}{s}$ – velocity of the stone, which was thrown up;

$V_2 = 8.97\frac{m}{s}$ –velocity of the stone, which was thrown down;

$H = 6.63m$ – height of the cliff;

h – height of the point where the stones cross paths (above the base of the cliff).

The equation of motion for the first stone (which was thrown up) respect to the Y-axis:

$$y_1 = V_1 t - \frac{gt^2}{2} \qquad (1)$$

The equation of motion for the second stone (which was thrown down) respect to the Y-axis:

$$y_2 = H - V_2 t - \frac{gt^2}{2} \qquad (2)$$

When stones the stones cross paths, their coordinates are equal:

$$y_2 = y_1 \qquad (3)$$

$(3)$and (2)in (1):

$$V_1 t_{cross} - \frac{g t_{cross}{}^2}{2} = H - V_2 t_{cross} - \frac{g t_{cross}{}^2}{2}$$

$$H = t_{cross}(V_2 + V_1)$$

$$t_{cross} = \frac{H}{V_2 + V_1} \qquad (4)$$

$$h = y_2(t_{cross}) = y_1(t_{cross}) = V_1 t_{cross} - \frac{g t_{cross}{}^2}{2} = \frac{V_1 H}{V_2 + V_1} - \frac{gH^2}{2(V_2 + V_1)^2} =$$

$$= \frac{H}{V_2 + V_1}\left(V_1 - \frac{gH}{2(V_2 + V_1)}\right) = \frac{6.63m}{8.97\frac{m}{s} + 8.97\frac{m}{s}}\left(8.97\frac{m}{s} - \frac{9.8\frac{m}{s^2}\cdot 6.63m}{2\left(8.97\frac{m}{s} + 8.97\frac{m}{s}\right)}\right)$$

$$= 2.65m$$

This means that paths will cross on a hight $h = 2.65m$ where each Y-coordinate is $2.65m$ ($y_2 = y_1 = 2.65m$).

The location (above the base of the cliff) of the point where the paths of the stones will cross:

$$h = 2.65\ m$$

Answer: paths will cross at the hight $\mathrm{h} = 2.65\ \mathrm{m}$ above the base of the cliff.