

Task. Write the equation of motions of simple harmonic oscillator which has an amplitude of 5 cm and it executes 150 oscillation in 5 minutes with an initial phase of 45 degree also obtain the value of its maximum velocity.

Solution. The equation of a simple harmonic oscillation has the following form

$$x(t) = A \cos(\omega t + \phi)$$

where A is the amplitude, ω is the angular frequency, and ϕ is the phase.

Its velocity is then given by the formula:

$$v(t) = x'(t) = (A \cos(\omega t + \phi))' = -A\omega \sin(\omega t + \phi).$$

Hence the maximum of velocity is equal to

$$\max v(t) = A\omega.$$

In our case $A = 5 \text{ cm}$, and $\phi = 45^\circ = \frac{\pi}{4}$. Also, by assumption the oscillator executes 150 oscillation in 5 minutes. So its frequency is

$$\nu = \frac{150}{5 \text{ min}} = \frac{150}{5 * 60 \text{ sec}} = \frac{150}{300 \text{ sec}} = 0.5 \text{ sec}^{-1},$$

whence the angular frequency is

$$\omega = 2\pi\nu = 2\pi * 0.5 \text{ sec}^{-1} = \pi \text{ rad/sec}.$$

Therefore the equation of motion for this oscillator is the following one:

$$x(t) = 5 \cos(\pi t + \frac{\pi}{4}) \text{ cm}$$

and the maximum of velocity is equal to

$$\max v(t) = A\omega = 5 \text{ cm} * \pi \text{ rad/sec} = 5\pi \text{ rad cm/sec}.$$

Answer. $x(t) = 5 \cos(\pi t + \frac{\pi}{4}) \text{ cm}$, $\max v = A\omega = 5\pi \text{ rad}\cdot\text{cm/sec}$.