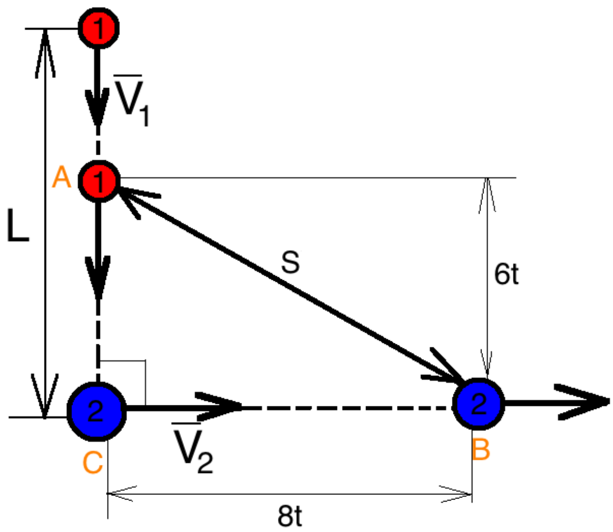
At noon a ship S1 is 20 miles north of a ship S2. If S1 is sailing south at a rate of 6 miles per hour and S2 is sailing east at a rate of 8 miles per hour, find the time when they are nearest together.

Solution:



Let t - the travel time in hours of both ships.
L = 20 - the distance between the ships at the beginning of the movement.
$V_1 = 6\,\frac{mile}{hour}$ - rate of the sheep S1
$V_2 = 6\,\frac{mile}{hour}$ - rate of the sheep S1

Then:
6t = travel dist of the S1 (sailing north toward the ref point C)
8t = travel dist of the S2 (sailing west from the ref point C)

The two ships course form a right triangle from the ref points A, B and C. $(\angle ACB = 90^o)$
The distance between the two ships, is the hypotenuse (S):

$$S^2 = (20 - 6t)^2 + (8t)^2$$
$$S^2 = 400 - 240t + 36t^2 + 64t^2$$
$$S^2 = 100t^2 - 240t + 400$$

distance between ships is minimum when the derivative $\frac{dS}{dt}$ is zero (local minimum of the function):

$$S = \sqrt{100t^2 - 240t + 400}$$
$$\frac{dS}{dt} = \frac{1}{2\sqrt{100t^2 - 240t + 400}} \cdot (200t - 240) = 0$$

$$200t - 240 = 0$$

$$t = \frac{240}{200} = 1.2\ hour$$

After the time $t = 1.2\ hour$ ships will be nearest together at a minimum distance:

$$S_{min} = S(1.2) = \sqrt{100 \cdot (1.2)^2 - 240 \cdot 1.2 + 400} = 16\ miles$$

Answer: after the time $t = 1.2\ hour$ ships will be nearest together at a minimum distance $S_{min} = 16\ miles$.