A uniform ladder rests against a rough wall so that it makes an angle of 60.0 degrees with the ground. the ladder is 10.0m long and weighs 150N. How far can a 250N man go before the ladder slips? The coefficient of friction between the ladder and the ground is 0.400; between the ladder and the wall is 0.450.

Solution:

$\mu_1 = 0.400 -$ coefficient of friction between the ladder and the ground ;

$\mu_2 = 0.450 -$ coefficient of friction between the ladder and the wall;

$\alpha = 60^o -$ angle *which* ladder makes with the ground

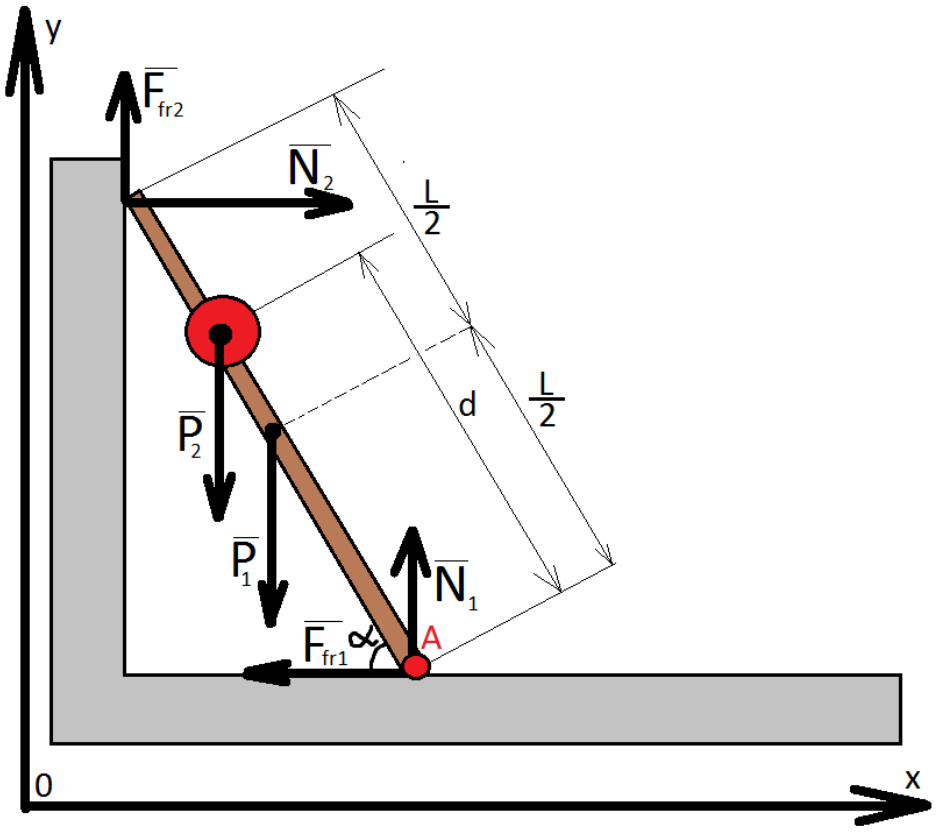
$N_1 - \ reaction\ force\ from\ the\ ground$

$N_2 - \ reaction\ force\ from\ the$ wall

$P_1 = 150N - \ weight\ of\ the\ ladder$

$P_2 = 250N - \ weight\ of\ the\ man$

$L = 10.0m - \ length\ of\ the\ ladder$



We will consider the extreme case when the person is standing at a maximum distance d from the beginning of the ladder.

Newton's second law for the ladder (the first law of equilibrium):

$$\overrightarrow{F_{fr1}} + \overrightarrow{F_{fr2}} + \overrightarrow{P_1} + \overrightarrow{P_2} + \overrightarrow{N_1} + \overrightarrow{N_2} = \vec{0}$$

Projection of the law on the X-axis:

$$x: N_2 - F_{fr1} = 0 \qquad (1)$$

Projection of the law on the X-axis:

$$y: N_1 + F_{fr2} - P_1 - P_2 = 0 \qquad (2)$$

Law of dry friction:

$$F_{fr1} = \mu_1 N_1 \qquad (3)$$

$$F_{fr2} = \mu_2 N_2 \qquad (4)$$

$(3) and (4) in\ (1) and\ (2)$:

$(3) \rightarrow (1): N_2 - \mu_1 N_1 = 0$

$$N_1 = \frac{N_2}{\mu_1} \qquad (5)$$

$(4) \rightarrow (2): N_1 + \mu_2 N_2 - P_1 - P_2 = 0$

$(5) \rightarrow (6): \frac{N_2}{\mu_1} + \mu_2 N_2 - P_1 - P_2 = 0$

$$N_2 = \frac{\mu_1 (P_1 + P_2)}{1 + \mu_1 \mu_2} = \frac{0.400 \cdot (150N + 250N)}{1 + 0.400 \cdot 0.450} = 135.6N$$

$$F_{fr2} = \mu_2 N_2 = 0.450 \cdot 135.6N = 61.02N$$

Momentum equation for point A (the second law of equilibrium):

$$A: M_{P_1} + M_{P_2} + M_{fr2} + M_{N_2} = 0 \qquad (6)$$

$(M_{N_1} = M_{fr1} = 0, because\ moment\ arm\ of\ this\ forces\ is\ zero)$

$$M_{P_1} = -P_1 \cdot \frac{L}{2} \cos\alpha \ \ (minus\ sign\ because\ of\ the\ direction\ of\ force)$$

$$M_{P_2} = -P_2 \cdot d \cos\alpha$$

$$M_{fr2} = F_{fr2} \cdot L \cos\alpha$$

$$M_{N_2} = N_2 \cdot L \sin\alpha$$

$\rightarrow (6)$:

$$F_{fr2} \cdot L \cos\alpha + N_2 \cdot L \sin\alpha - P_2 \cdot d \cos\alpha - P_1 \cdot \frac{L}{2} \cos\alpha = 0$$

$$d = \frac{F_{fr2} \cdot L \cos\alpha + N_2 \cdot L \sin\alpha - P_1 \cdot \frac{L}{2} \cos\alpha}{P_2 \cos\alpha} =$$

$$= \frac{61.02N \cdot 10m \cdot \frac{1}{2} + 135.6N \cdot 10m \cdot \frac{\sqrt{3}}{2} - 150N \cdot \frac{10m}{2} \cdot \frac{1}{2}}{250N \cdot 0.5} = 8.835m$$

Answer: man can go up the stairs a distance of $d = 8.835m$ before the ladder slips.