Task. When brakes are applied the speed of a train decreases from $v_0 = 96\ km/h$ to $v_1 = 48\ km/h$ in $d_0 = 800\ m$. How much further will the train moves before coming to rest?

Solution. Suppose that the train moves with constant deceleration $a < 0$. If the brakes were applied at time $t = 0$, then at time t the train will pass the distance

$$d(t) = v_0 t + \frac{at^2}{2}$$

and the velocity of the train at that time is

$$v(t) = v_0 + at.$$

Thus we have a system of the following two equations:

$$\begin{cases} d_0 = v_0 t + \frac{at^2}{2} \\ v_1 = v_0 + at \end{cases}$$

Let us first find t and a for which $d(t) = d_0$ and $v(t) = v_1$.

From the second equation we get:

$$at = v_1 - v_0.$$

Substituting it into the first equation we obtain

$$d_0 = v_0 t + \frac{at \cdot t}{2} = v_0 t + \frac{(v_1 - v_0)t}{2} = \frac{2v_0 t + v_1 t - v_0 t}{2} = \frac{(v_0 + v_1)t}{2}.$$

Hence

$$t = \frac{2d_0}{v_0 + v_1}.$$

Substituting this expression for t into the second equation we obtain the formula for a:

$$a = \frac{v_1 - v_0}{t} = \frac{(v_1 - v_0)(v_0 + v_1)}{2d_0} = \frac{v_1^2 - v_0^2}{2d_0}.$$

Now we will find the time $\bar{t}$, when the train will stop. It can be derived from the relation

$$v(\bar{t}) = 0.$$

From the second equation we have

$$0 = v(\bar{t}) = v_0 + a\bar{t}$$

whence

$$\bar{t} = -\frac{v_0}{a} = -\frac{2d_0 v_0}{v_1^2 - v_0^2} = \frac{2d_0 v_0}{v_0^2 - v_1^2}.$$

Hence the total distance passed by the train after applying brakes is

$$d(\bar{t}) = v_0 \bar{t} + \frac{a\bar{t}^2}{2}.$$

Taking to account that

$$a\bar{t} = -v_0,$$

and using the formula for $\bar{t}$, we obtain that

$$d(\bar{t}) = v_0 \bar{t} + \frac{a\bar{t} \cdot \bar{t}}{2} = v_0 \bar{t} - \frac{v_0 \bar{t}}{2} = \frac{v_0 \bar{t}}{2} = \frac{d_0 v_0^2}{v_0^2 - v_1^2}.$$

Therefore after passing d_0 the train will move to the rest the distance

$$d_1 = d(\bar{t}) - d_0 = \frac{d_0 v_0^2}{v_0^2 - v_1^2} - d_0 = \frac{d_0 v_0^2}{v_0^2 - v_1^2} - \frac{d_0(v_0^2 - v_1^2)}{v_0^2 - v_1^2} = \frac{d_0 v_1^2}{v_0^2 - v_1^2}.$$

Substituting values we obtain:

$$d_1 = \frac{d_0 v_1^2}{v_0^2 - v_1^2} = \frac{0.8 \cdot 48^2}{96^2 - 48^2} \approx 0.267\ km = 267\ m.$$

Answer. 267 m.