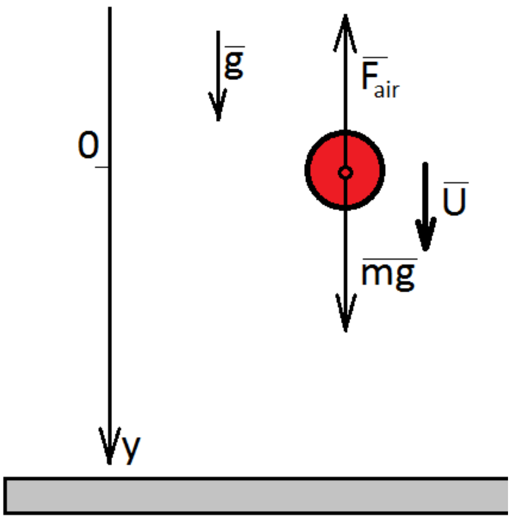
A skydiver of mass m jumps from an aircraft and falls towards the ground. There are two principal forces acting upon her: gravity mg pulling her down and air resistance slowing her fall. Assume that the upward force of air resistance is proportional to the square of her speed v, so that we can write air resistance as equal to kv2, where k is a constant independent of v. Thus, the faster she falls, the greater the air resistance.

(a) (12 marks) After the skydiver has been falling for some time, air resistance brings her acceleration to zero, so that she is now falling at constant speed u. Derive a formula for u in terms of m, g and k.

(b) (8 marks) Derive the dimensions of the constant k in terms the fundamental dimensions of mass M, length L and time t

Solution:

The condition that the air resistance brings her acceleration to zero:



Newton's second law for skydiver: $m\vec{a} = \overrightarrow{mg} + \overrightarrow{F_{air}}$

$$x: ma = mg - F_{air}$$

$$a = 0\ (acceleration\ is\ zero) \rightarrow$$

$$mg - F_{air} = 0\ (1)$$

$$F_{air} = kU^2\ (2)$$

$$(1) in\ (2): mg = kU^2$$

$$U = \sqrt{\frac{mg}{k}}$$

We can take the above equation and solve it for "k". Then we will take all the quantities on the right-hand-side and substitue M's, L's and T's as appropriate:

$$M = m\ (mass);\ \ V = \frac{L}{T}\ (speed); g = \frac{L}{T^2}\ (acceleration)$$

$$mg = kU^2$$

$$k = \frac{mg}{U^2} = \frac{M\frac{L}{T^2}}{\frac{L^2}{T^2}} = \frac{MLT^2}{L^2T^2} = \frac{M}{L}$$

Answer: a) $U = \sqrt{\frac{mg}{k}}$

b) $k = \frac{M}{L}$