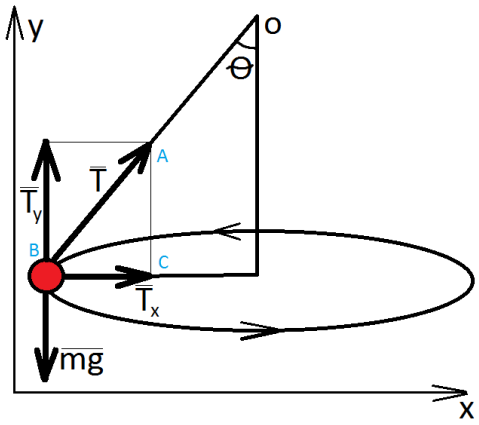
A ball of mass 0.1 g is suspended by a string of length 30 cm , keeping it always taut , it describes a circle of radius 15 cm . find the angular speed of the ball.

Solution:

Situation of the problem is shown on the figure:



$$\sin\theta = \frac{15\ cm}{30\ cm} = \frac{1}{2}\ or\ \theta = 30^o$$

Right triangle ABC:

$T_x = T\sin\theta$; $T_x = T\cos\theta$;

Newton's second law for the ball:

$$\vec{T} + \overrightarrow{mg} = m\vec{a}$$

The projection on the X-axis:

$$x: T\sin\theta = ma \qquad (1)$$

centripetal acceleration of the ball (circular movement)

$$a = \frac{V^2}{r} = \omega^2 r \qquad (2)$$

$$(2) in\ (1): T\sin\theta = m\omega^2 r \qquad (3)$$

The projection on the Y-axis:

$$y: T\cos\theta - mg = 0$$

$$T\cos\theta = mg \qquad (4)$$

$$(3) \div (4): \frac{T\sin\theta}{T\cos\theta} = \frac{m\omega^2 r}{mg}$$

$$\tan\theta = \frac{\omega^2 r}{g}$$

$$\omega^2 = \frac{g\cdot\tan\theta}{r}$$

$$\omega = \sqrt{\frac{g\cdot\tan\theta}{r}} = \sqrt{\frac{9.8\frac{m}{s^2}\cdot\tan 30^o}{0.15m}} = 6.14\ \frac{rad}{s}$$

Answer: angular speed of the ball is $\omega = 6.14\ \frac{rad}{s}$