Task. Two straight line drawn on same displacement-time graph and make 30 degree and 60 degree with time axis. Which represent greater velocity also find ratio of velocity?

Solution. We have two graphs of displacements functions $d_1(t)$ and $d_2(t)$. By assumption both of them are straight lines, so

$$d_1(t) = k_1 t + b_1, \qquad d_2(t) = k_2 t + b_2$$

for some numbers k_1, b_1, k_2, b_2. Moreover, the first line constitute 30 degree with time axis, and the second one constitute 60 degree.

It is well-known that in teh equaltion of straight line $d(t) = kt + b$ the coefficient k is equal to tangent of the angle α between the line and time axis:

$$k = \tan \alpha.$$

Hence for the first line

$$k_1 = \tan 30° = \frac{1}{\sqrt{3}},$$

and for the second line

$$k_2 = \tan 60° = \sqrt{3}.$$

Now notice that the velocity of an object is the time derivative of its displacement:

$$v(t) = d'(t).$$

For the linear function $d(t) = kt + b$ its derivative is equal to the coefficient k:

$$d'(t) = (kt + b)' = k.$$

Thus for the first object its velocity is

$$v_1(t) = k_1 = \frac{1}{\sqrt{3}} \ m/s,$$

and for the second one:

$$v_2(t) = k_2 = \sqrt{3} \ m/s.$$

Therefore the second velocity is greater the the first one:

$$v_1 < v_2.$$

Their ratio is

$$\frac{v_2}{v_1} = \frac{\sqrt{3}}{1/\sqrt{3}} = \sqrt{3} \cdot \sqrt{3} = 3.$$

Answer. $v_2 = 3v_1$.