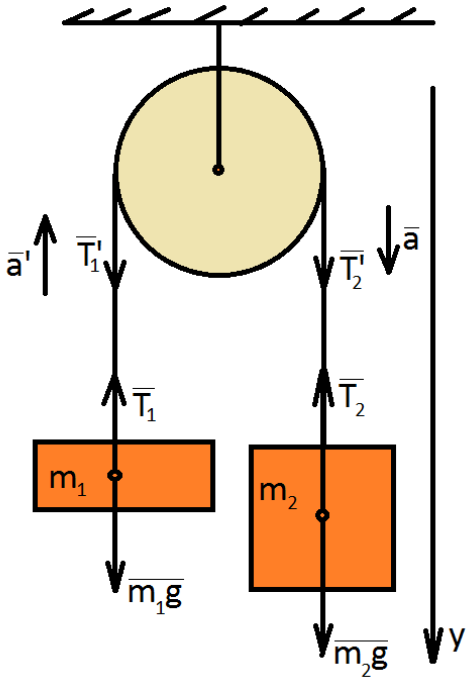
An Atwood machine consist of a single pulley about an axis through it centre and perpendicular to its plane. The length of the inextensible string connecting the two masses and going over the pulley is L. Calculate the acceleration of the system.

Solution:

Forces acting on the two masses and pulley:

mg – the force of gravity, T – string tension force, a – acceleration of the system;



Tension force to each end of the string:

$$|\overrightarrow{T_1}| = |\overrightarrow{T'_1}|;\ |\overrightarrow{T_2}| = |\overrightarrow{T'_2}|;|\overrightarrow{a'}| = |\vec{a}|$$

If we consider that we have ideal pulley:

$$\overrightarrow{T'_1} = \overrightarrow{T'_2}$$

Newton's second law for the first mass:

$$\overrightarrow{T_1} + \overrightarrow{m_1 g} = m_1\vec{a}$$

$$y: T_1 - m_1 g = m_1 a$$

$$T_1 = m_1 a + m_1 g\ (1)$$

Newton's second law for the second mass:

$$\overrightarrow{T_2} + \overrightarrow{m_2 g} = m_1\vec{a}$$

$$y: m_2 g - T_2 = m_2 a$$

$$T_2 = m_2 g - m_2 a = T_1(from\ (1))\ (2)$$

$$(1) = (2): m_2 g - m_2 a = m_1 a + m_1 g$$

$$a(m_1 + m_2) = g(m_2 - m_1)$$

$$a = \frac{g(m_2 - m_1)}{m_1 + m_2};$$

We do not know the values of the masses, so we can take the absolute difference $|m_2 - m_1|$ for the positive value of the acceleration:

$$a = \frac{g|m_2 - m_1|}{m_1 + m_2}$$

Answer: $a = \frac{g|m_2 - m_1|}{m_1 + m_2}$