When a volcano erupts, it emits smoke and lava in all direction with a velocity of 280m/s. The height of volcano is 5 km and there is a house located at a distance of 8 km from the centre of the mountain. 5s after an eruption, a group of people start fleeing from the house in a car that can run at a maximum speed of 90km/h.

Assuming that they travel in a straight line away from the volcano, what additional distance should they have covered to escape the fireball?

Solution:

$h = 5km = 5000m -$ height of volcano;

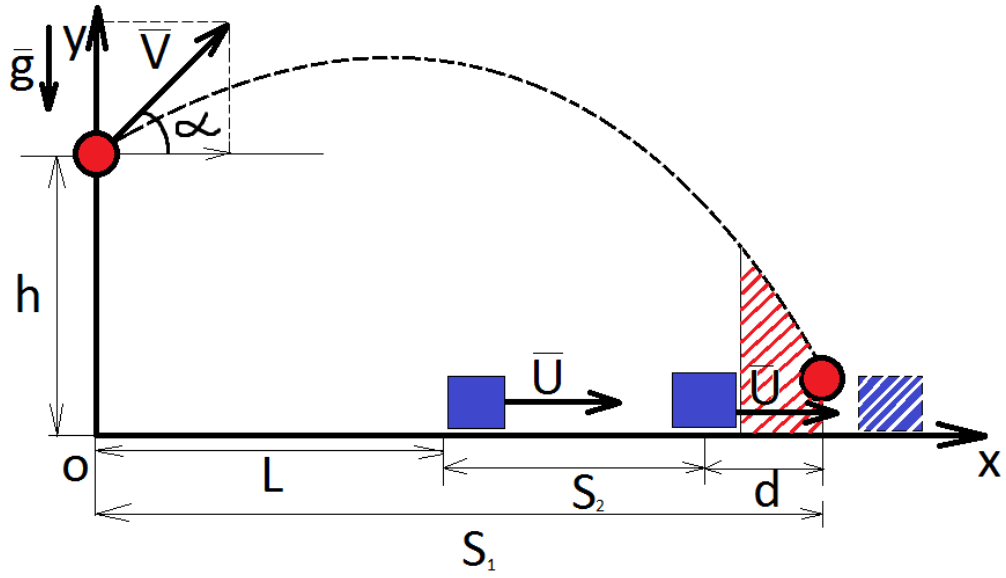
$T = 5s\ time\ after$ eruption, a group of people start fleeing from the house in a car;

$V = 280\frac{m}{s} - velocity\ of\ the$ lava;

$U = 90\frac{km}{h} = 25\frac{m}{s} -$ maximum speed of the car;

$L = 8km = 8000m -$ a distance from the centre of the mountain;

$d -$ additional distance should group have covered to escape the fireball.



We need to consider the fireball, which emitted from volcano at an angle of 45 degrees, because at this angle you can find the maximum distance that the lava can reach:

Equation of motion of fireball: ($\alpha = 45^o$)

$$y: 0 = h + V_y t - \frac{gt^2}{2}; where\ t -\ time\ before\ falling\ to\ the\ ground$$

$$x: S_1 = V_x t$$

$$V_y = V\sin\alpha\ ;\ V_x = V\cos\alpha \rightarrow$$

$$h = \frac{gt^2}{2} - Vt\sin\alpha\ (1)$$

$$S_1 = Vt\cos\alpha$$

$$t = \frac{S_1}{V\cos\alpha}(2)$$

$$(2) in\ (1)$$

$$h = \frac{g\left(\frac{S_1}{V\cos\alpha}\right)^2}{2} - \frac{S_1}{\cos\alpha}\sin\alpha$$

$$h = \frac{gS_1^2}{2V^2\cos^2\alpha} - S_1\tan\alpha$$

Substitute numerical values to find the maximum distance S_1: $(\cos\alpha = \sin\alpha = \frac{1}{\sqrt{2}}; \tan\alpha = 1)$

$$5000 = \frac{9.8S_1^2}{\left(280\frac{m}{s}\right)^2} - S_1$$

$$0.000125 \cdot S_1^2 - S_1 - 5000 = 0$$

We have a quadratic equation whose roots are:

$$S_1 = -3483.31; 11483.3$$

$$S_1 > 0, distance\ can\ not\ be\ negative\ \rightarrow S_1 = 11483.3m$$

$$t = \frac{S_1}{V\cos\alpha} = \frac{11483.3m}{280\frac{m}{s}\cdot\frac{1}{\sqrt{2}}} = 58s$$

Now we can consider the motion of the car. Distance that the car has passed:

$$S_2 = U\cdot(t-T) = 25\frac{m}{s}(t-5s) = 25\frac{m}{s}(58s-5s) = 1325m$$

With $S_1\ and\ S_2$ we can find additional distance should group have covered to escape the fireball:

$$d = S_1 - L - S_2$$

$$d = 11483.3m - 8000m - 1325m = 2158m$$

Answer: additional distance should group have covered to escape the fireball: $d = 2158m$.