

a projectile has a max height of 20 m and a range of 30 m. what is the initial velocity?

Solution:

Equations related to trajectory motion (projectile motion) are given by:

$$\text{max height: } h = \frac{v_0^2 \sin^2 \theta}{2g}$$

$$\text{range: } R = \frac{v_0^2 \sin 2\theta}{g} = \frac{2v_0^2 \sin \theta \cos \theta}{g}$$

$$\text{From the first equation } \sin^2 \theta = \frac{2gh}{v_0^2} \text{ or } \sin \theta = \sqrt{2 \frac{gh}{v_0^2}}$$

$$\text{From the second equation: } \cos \theta = \frac{gR}{2v_0^2 \sin \theta} = \frac{gR}{2v_0^2 \sqrt{2 \frac{gh}{v_0^2}}} = \sqrt{\frac{gR}{8v_0^2}}$$

Pythagorean trigonometric identity:

$$\sin^2 \theta + \cos^2 \theta = 1$$

Therefore:

$$2 \frac{gh}{v_0^2} + \frac{gR}{8v_0^2} = 1$$

$$v_0 = \sqrt{2g \left(h + \frac{R}{16} \right)} = 20.7 \frac{m}{s}$$

Answer: $20.7 \frac{m}{s}$