

The acceleration of a body moving along x-axis is given by  $a=4x^3$ , where a is in  $\text{m/s}^2$  and x is in metre. If at  $x=0$  the velocity of body is 2m/s, then find its velocity at  $x=4$  m.

$$a = 4x^3$$

By definition:

$$a = \frac{dv}{dt} = \frac{dv}{dx} \frac{dx}{dt} = \frac{dv}{dx} v$$

Separate the variables:

$$4x^3 dx = v dv$$

Integrate:

$$x^4 + C = \frac{v^2}{2}$$

$$v(x) = \pm \sqrt{2} \sqrt{x^4 + C}$$

Initial condition:  $v(0) = 2$

$$v(0) = \sqrt{2} \sqrt{0^4 + C} = 2$$

$$C = 2$$

Therefore,

$$v(x) = \sqrt{2} \sqrt{x^4 + 2}$$

Finally,

$$v(4) = \sqrt{2} \sqrt{4^4 + 2} = 2\sqrt{129} \frac{m}{s}$$

$$\text{Answer: } v(4) = 2\sqrt{129} \frac{m}{s}$$