

An object moves in a quadrant of radius R from point A to B such that A & B lie on two ends of the quadrant & the force applied is always directed towards B. Find the work done.

Solution:

Formula for work:

$$A = \int_A^B \vec{F} \cdot \vec{ds} = \int_A^B F \cdot ds \cdot \cos \alpha \cdot d\alpha \quad (1)$$

$$ds = R \cdot \alpha$$

For a small angle alpha:

$$\alpha \approx \sin \alpha$$

$$ds = R \sin \alpha \quad (2)$$

(2) in (1):

$$A = \int_A^B FR \sin \alpha \cdot \cos \alpha \, d\alpha = \int_A^B FR \cos \alpha \, d(\cos \alpha) = \frac{FR \cos^2 \alpha}{2} \Big|_A^B =$$

$$= \frac{FR \cos^2 \alpha}{2} \Big|_{\frac{\pi}{4}}^0 = \frac{FR \cos^2 0}{2} - \frac{FR \cos^2 45^\circ}{2} =$$

$$= \frac{FR}{2} - \frac{FR}{4} = \frac{FR}{4} \quad (F - \text{force}, R - \text{radius})$$

Answer: $A = \frac{FR}{4}$

