

An object moves in a quadrant of radius R from point A to B such that A & B lie on two ends of the quadrant & the force applied is always directed towards B. Find the work done.

Solution:

Formula for work:

$$A = \int_A^B \vec{F} * \vec{ds} = \int_A^B F * ds * \cos \alpha * d\alpha \quad (1)$$

$$ds = R * \alpha$$

For a small angle alpha:

$$\alpha \approx \sin \alpha$$

$$ds = R \sin \alpha \quad (2)$$

(2) in (1):

$$\begin{aligned} A &= \int_A^B FR \sin \alpha * \cos \alpha d\alpha = \int_A^B FR \cos \alpha d(\cos \alpha) = \frac{FR \cos^2 \alpha}{2} \Big|_A^B = \\ &= \frac{FR \cos^2 \alpha}{2} \Big|_{\frac{\pi}{4}}^0 = \frac{FR \cos^2 0}{2} - \frac{FR \cos^2 45^\circ}{2} = \\ &= \frac{FR}{2} - \frac{FR}{4} = \frac{FR}{4} \quad (F - \text{force}, R - \text{radius}) \end{aligned}$$

Answer: $A = \frac{FR}{4}$

