Obtain an expression for the time period of a satellite orbiting the earth. A space shuttle is in a circular orbit at a height of 250 km from the earth's surface, where the acceleration due to earth's gravity is 0.93 g. Calculate the period of its orbit. Take g = 9.8 ms−2 and the radius of the earth R = 6.37 × 106m.

Solution:

Let T- the time period of a satellite orbiting the earth.
h = 250 km = 250 000 m = $2.5*10^5$ m – height from earth's surface
R = 6.67 10^6 m - the radius of the earth

$$v - velocity\ of\ a\ satellite$$

$$T = \frac{2\pi(R+h)}{v}$$

So we find the velocity of satellite.
If m – mass of satellite and M-mass of Earth and $\gamma - gravitational\ constant$

$$F = \gamma mM/((R+h)^2)$$

In other hand $F = ma = m\frac{v^2}{R+h} -> v = \sqrt{\gamma\frac{M}{R+h}} = \sqrt{\frac{\gamma M}{R^2}\frac{R^2}{R+h}} = \sqrt{g\frac{R^2}{R+h}}$

So

$$T = \frac{2\pi(R+h)}{v} = \frac{2\pi(R+h)}{\sqrt{g\frac{R^2}{R+h}}} = \frac{2\pi(R+h)^{3/2}}{R\sqrt{g}} = \frac{2*3.14*(6.67*10^6+2.5*10^5)^{\frac{3}{2}}}{6.67*10^6\sqrt{9.81}} = 5472\ seconds$$
$$= 1.52\ hour$$

Answer: 1.52 hours