Derive an expression relating impulse and linear momentum. In a safety test, a car of mass 1000 kg is driven into a brick wall. Its bumper behaves like a spring (k = 5 × 106 Nm−1) and is compressed by a distance of 3 cm as the car comes to rest. Determine the initial speed of the car.

Impulse $\vec{J}$ produced from time t_1 to t_2 is defined to be:

$$\vec{J} = \int_{t_1}^{t_2} \vec{F} dt$$

where $\vec{F}$ is the force applied from t_1 to t_2
From Newton's second law, force is related to momentum p by:

$$\vec{F} = \frac{d\vec{p}}{dt}$$

Therefore:

$$\vec{J} = \int_{t_1}^{t_2} \vec{F} dt = \int_{t_1}^{t_2} \frac{d\vec{p}}{dt} dt = \int_{t_1}^{t_2} d\vec{p} = \Delta\vec{p}$$

The law conservation of energy:

$$\frac{mv^2}{2} = \frac{k\Delta l^2}{2}$$

m – mass of car

v - initial speed of car

Δl – deformation of bumper

Therefore:

$$v = \sqrt{\frac{k\Delta l^2}{m}} = \sqrt{\frac{5\ 10^6 \frac{N}{m}\ (0.03\ m)^2}{1000\ kg}} = 2.12 \frac{m}{s}$$

Answer: the initial speed of the car equals $2.12 \frac{m}{s}$