A toy cannon uses a spring to project a 5.39-g soft rubber ball. The spring is originally compressed by 4.99 cm and has a force constant of 8.03 N/m. When the cannon is fired, the ball moves 14.9 cm through the horizontal barrel of the cannon, and the barrel exerts a constant friction force of 0.031 3 N on the ball. At what point does the ball have maximum speed?

The law of conservation of energy:

$T_1 + U_1 = T_2 + U_2 + A$

$T = \frac{mv^2}{2}$ - kinetic energy, m - mass of the body, v - speed

$U = \frac{kx^2}{2}$ - potential energy, k- force constant of spring, x – deformation

$A = Fl$ – work of constant friction force , F – magnitude of force, l – position of ball.

So, for our case we have:

$\frac{kx_0^2}{2} + 0 + 0 = \frac{mv^2}{2} + \frac{k(x_0 - x)^2}{2} + Fx$ (for $x < x_0$, if $x \geq x_0$, ball will slow down)

x_0 – initial spring deformation, x – current position of ball

$$\frac{v^2}{2} = -\frac{k}{2m}x^2 + \left(\frac{k}{m}x_0 - \frac{F}{m}\right)x$$

Maximum value at point $\frac{dv}{dx} = 0$:

$$-\frac{k}{m}x_{max} + \frac{k}{m}x_0 - \frac{F}{m} = 0$$

$$x_{max} = x_0 - \frac{F}{k} = 4.99\text{ cm} - \frac{0.0313\text{N}}{0.0803\frac{\text{N}}{\text{cm}}} = 4.99\text{cm} - 0.39\text{cm} = 4.6\text{ cm}$$

Answer: 4.6 cm from initial point