Two wires P ad Q, each of the same length and same material, are connected in parallel to a battery. The diameter of P is half that of Q. What fraction of the total current passes through P?

A. 0.2

B. 0.25

C. 0.33

D. 0.5

The resistance R of a conductor of uniform cross section can be computed as

$$R = \frac{\rho l}{A} = \frac{\rho l}{\pi d^2/4}$$

where l is the length of the conductor, measured in metres [m], $A = \frac{\pi d^2}{4}$ is the cross-section area of the conductor measured in square metres [m²], and ρ (rho) is the electrical resistivity (also called specific electrical resistance) of the material, measured in ohm-metres (Ω·m).

Therefore:

$$R_P = \frac{\rho l}{\pi d_P^2/4}$$

$$R_Q = \frac{\rho l}{\pi d_Q^2/4}$$

Or:

$$\frac{R_P}{R_Q} = \frac{d_Q^2}{d_P^2} = 4$$

Ohm's law states that the current through a conductor between two points is directly proportional to the potential difference across the two points:

$$I = \frac{V}{R}$$

where I is the current through the conductor in units of amperes, V is the potential difference measured across the conductor in units of volts, and R is the resistance of the conductor in units of ohms.

Therefore, current passes through P equals:

$$I_P = \frac{V}{R_P}$$

Current passes through Q equals:

$$I_Q = \frac{V}{R_P}$$

Total current:

$$I = I_Q + I_P = V\left(\frac{1}{R_Q} + \frac{1}{R_P}\right)$$

Fraction of the total current passes through P:

$$\frac{I_P}{I} = \frac{\frac{V}{R_P}}{V\left(\frac{1}{R_Q} + \frac{1}{R_P}\right)} = \frac{1}{\frac{R_P}{R_Q} + 1} = \frac{1}{5} = 20\ \%$$

Answer: $20\ \%$