Calculate the average energy per revolution and find energy by electron in a betatron to which is applied a maximum magnetic field of 0.5 tesla operating in a stable orbit of diameter.

The particles have maximum energy when the magnetic field is at its strongest value but the formula used for the cyclotron will not work for betatrons because the electron will be relativistic. However, if the total energy is much greater than the rest energy then $E = pc$ is a good approximation. As the centripetal force is again provided by the Lorentz force

$$\frac{mv^2}{R} = qvB$$

m – mass of electron, B - maximum magnetic field, q – charge of electron, R=D/2 – radius of orbit

the maximum momentum will be:

$$p = RqB$$

therefore:

$$E = RqcB$$

As the electron accelerates it radiates energy at rate given by the Larmor formula in the rest frame of the electron:

$$\frac{dE}{dt} = -\frac{2e^2(\ddot{p})^2}{3c^3} = -\frac{2e^4\gamma^2B^2}{3m^2c^3}$$

$$\gamma = \frac{E}{m}$$

Or $\Delta E = \frac{2e^4\gamma^2B^2}{3m^2c^3}\Delta t$

Time of one revolution equals:

$$\Delta t = \frac{2\pi R}{v} = \frac{2\pi R}{RqB}m = \frac{2\pi m}{qB}$$

Therefore, average energy loss per revolution equals:

$$\Delta E = \frac{2e^4\left(\frac{RqcB}{m}\right)^2 B^2}{3m^2c^3}\frac{2\pi m}{qB} = \frac{4\pi e^4R^2qB^3}{3m^3c}$$