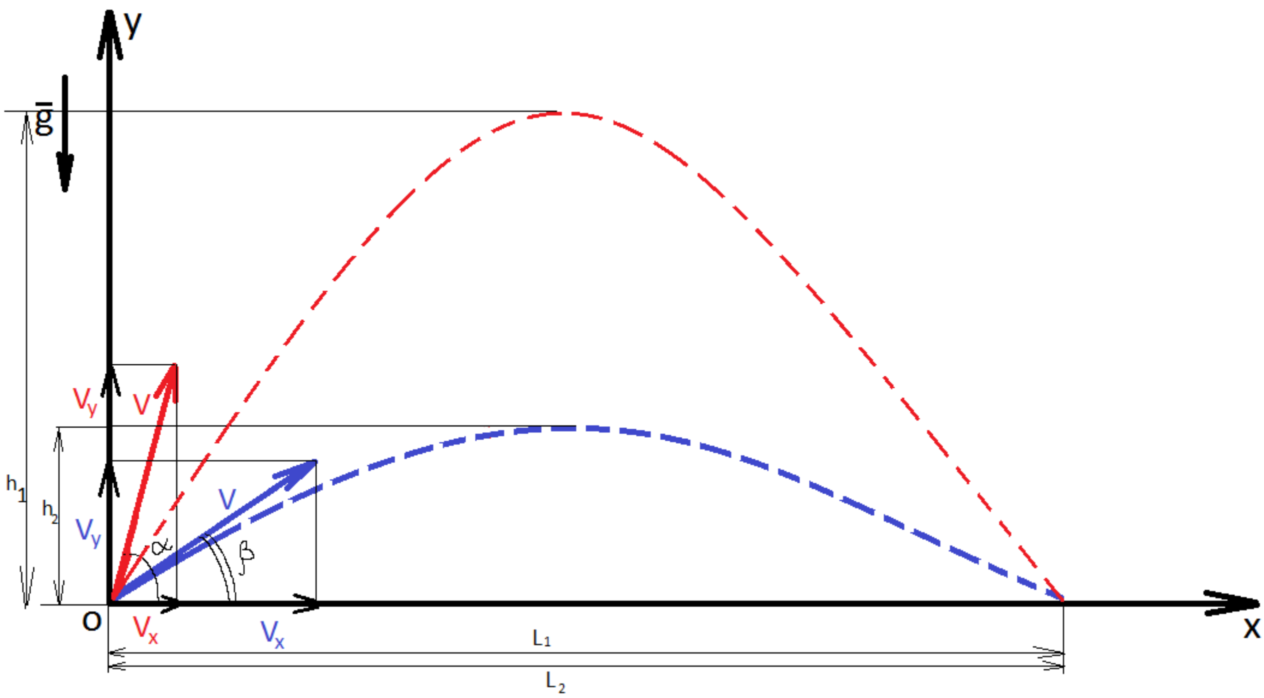
a body is projected at two complementary angles with same speed if maximum height reached by it are 40m and 90m find maximum range

**Solution:**

We can write the equations of motion for each of the bodies and use fact that:

$\alpha = 90^o - \beta$




**1.**

Equations for body, thrown at angle $\alpha$: ($L_1$ - maximum range of this body)

$$V_x = V\cos\alpha;\ V_y = V\sin\alpha;$$

$$x: L_1 = Vt\cos\alpha\ \ (1), t - \ \text{time of the flight}$$

$$y: 0 = Vt\sin\alpha - \frac{gt^2}{2}$$

$$V\sin\alpha = \frac{gt}{2}$$

$$t = \frac{2V\sin\alpha}{g}\ (2)$$

$$(2) in\ (1): \boldsymbol{L_1} = V\frac{2V\sin\alpha}{g}\cos\alpha = \frac{2V^2\sin\alpha\cos\alpha}{g}$$

Maximum height: the time taken to reach the maximum height is equal to half of the time of flight:

$$t_1 = \frac{t}{2} = \frac{V\sin\alpha}{g}$$

$$y: (half\ of\ the\ flight): h_1 = Vt_1 \sin\alpha - \frac{gt_1{}^2}{2}$$

$$h_1 = Vt_1 \sin\alpha - \frac{gt_1{}^2}{2}$$

$$\boldsymbol{h_1} = V\frac{V\sin\alpha}{g}\sin\alpha - \frac{g\left(\frac{V\sin\alpha}{g}\right)^2}{2} = \frac{V^2\sin^2\alpha}{2g}$$

Equations for body, thrown at angle $\beta = 90^o - \alpha$;

We will use the properties of trigonometric:

$\sin(90^o - \alpha) = \cos\alpha; \cos(90^o - \alpha) = \sin\alpha$

**2.**

Equations for body, thrown at angle $\alpha$: ($L_1$ - maximum range of this body)

$$V_x = V\cos\beta = V\sin\alpha\,;\ V_y = V\sin\beta = \mathrm{V}\cos\alpha\,;$$

$$x: L_1 = Vt'\sin\alpha\ \ (1)', t' - \text{ time of the flight}$$

$$y: 0 = Vt'\cos\alpha - \frac{gt^2}{2}$$

$$V\cos\alpha = \frac{gt'}{2}$$

$$t' = \frac{2V\cos\alpha}{g}\ (2)'$$

$$(2) in\ (1): \boldsymbol{L_2} = V\frac{2V\sin\alpha}{g}\cos\alpha = \frac{2V^2\sin\alpha\cos\alpha}{g} = \boldsymbol{L_1}$$

It means that the maximum range in both cases are equal.

Maximum height: the time taken to reach the maximum height is equal to half of the time of flight:

$$t_2 = \frac{t'}{2} = \frac{V\cos\alpha}{g}$$

$$y: (half\ of\ the\ flight): h_2 = Vt_2\cos\alpha - \frac{gt_2{}^2}{2}$$

$$h_2 = Vt_2\cos\alpha - \frac{gt_2{}^2}{2}$$

$$\boldsymbol{h_2} = V\frac{V\cos\alpha}{g}\cos\alpha - \frac{g\left(\frac{V\cos\alpha}{g}\right)^2}{2} = \frac{V^2\cos^2\alpha}{2g}$$

So, we have a system of equations:

$$\begin{cases} L = \dfrac{2V^2 \sin\alpha \cos\alpha}{g} \ (3) \\ h_1 = \dfrac{V^2 \sin^2\alpha}{2g} \ (4) \\ h_2 = \dfrac{V^2 \cos^2\alpha}{2g} \ (5) \end{cases}$$

$$\frac{(4)}{(5)}: \frac{h_1}{h_2} = \frac{V^2 \sin^2\alpha}{2g} * \frac{2g}{V^2 \cos^2\alpha} = \frac{\sin^2\alpha}{\cos^2\alpha} = \frac{40m}{90m} = \frac{4}{9}$$

apply the square root of both sides of the equation:

$$\frac{\sin^2\alpha}{\cos^2\alpha} = \frac{4}{9} => \frac{\sin\alpha}{\cos\alpha} = \frac{2}{3} = \tan\alpha;$$

$$\frac{(3)}{(4)}: \frac{L}{h_1} = \frac{2V^2 \sin\alpha \cos\alpha}{g} * \frac{2g}{V^2 \sin^2\alpha}$$

$$\frac{L}{h_1} = \frac{2\cos\alpha}{1} * \frac{2}{\sin\alpha} = \frac{4}{\tan\alpha};$$

$$\frac{2}{3} = \tan\alpha\,; =>$$

$$L = \frac{4h_1}{\tan\alpha} = \frac{3 * 4 * 40m}{2} = 240m$$

**Answer:** maximum range $L = 240m$