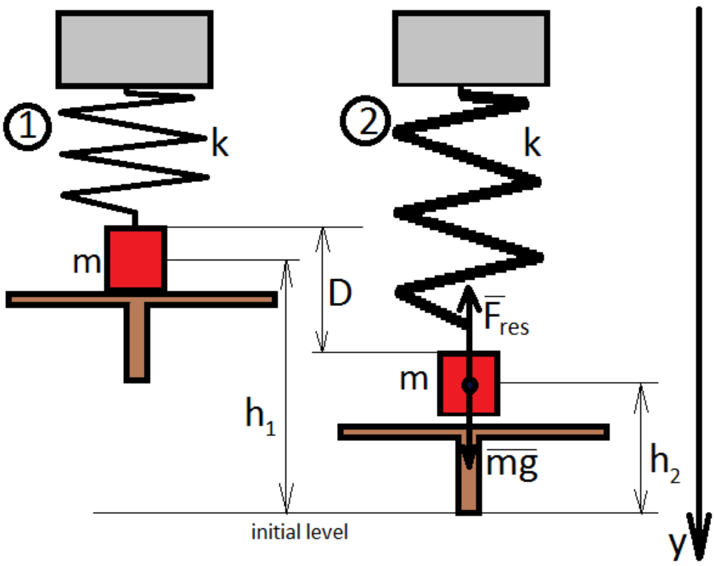
A weight of mass M is attached to the bottom end of a suspended spring. Initially the weight is supported with a board so that the spring is at its natural length. Next the board is gently lowered. When the spring is extended distance D, the weight separates from the board and come to rest. Let the total mechanical energy of the spring and weight be defined as U1 and for the state after the weight separates from the board as U2. What is (U2-U1)? Why is the answer (1/2)mgd?

Solution:



For the initial level of the system we will take the position of the board after the weight hanging on a spring, k- spring constant;

Mechanical energy in the first position:

$$E_{m1} = U_1 = E_{spring1} + E_{\text{weight1}},$$

$$E_{spring1} and\ E_{\text{weight1}}\ - the\ potential\ energy\ of\ the\ spring\ and\ the\ \text{weight}$$

$$E_{spring1} = 0\ (spring\ in\ resting\ position)$$

$$E_{\text{weight1}} = mgh_1 =>$$

$$U_1 = mgh_1\ (1)$$

Mechanical energy in the second position (weight at rest position):

$$E_{m2} = U_2 = E_{spring2} + E_{\text{weight2}}$$

$$E_{spring2} and\ E_{\text{weight2}}\ - the\ potential\ energy\ of\ the\ spring\ and\ the\ \text{weight}$$

$$E_{spring2} = \frac{kD^2}{2}\ (spring\ is\ stretched\ a\ distance\ D)$$

$$E_{\text{weight2}} = mgh_2 =>$$

$$U_2 = \frac{kD^2}{2} + mgh_2\ (2)$$

Newton's second law for a weight when it broke away from the board (the spring is stretched on length D):

$$\overrightarrow{F_{res}} + \overrightarrow{mg} = \vec{0}$$

$$y: -F_{res} + mg = 0\ (3)$$

Hooke's law:

$$F_{res} = kD (4)$$

$$(4) in\ (3): mg = kD$$

$$k = \frac{mg}{D}\ (5)$$

U_2-U_1:

$$(2) - (1): U_2 - U_1 = \frac{kD^2}{2} + mgh_2 - mgh_1\ (6)$$

$$(5) in\ (6): U_2 - U_1 = \frac{mgD^2}{2D} + mg(h_2 - h_1)$$

$$U_2 - U_1 = \frac{mgD}{2} + mg(h_2 - h_1)$$

$$h_1 - h_2 = D$$

$$U_2 - U_1 = \frac{mgD}{2} - mgD = -\frac{mgD}{2}$$

Answer: $U_2 - U_1 = -\frac{mgD}{2}$