

Question 32898

Let us first write parametric equations for uniform circular motion.

$$x(t) = R \cos(\omega t); y(t) = R \sin(\omega t) .$$

The x and y projections of velocity are:

$$v_x = x'(t) = -R\omega \sin(\omega t); v_y = y'(t) = R\omega \cos(\omega t) .$$

The absolute value of velocity is $v = \sqrt{v_x^2 + v_y^2} = \omega R$.

The x and y projections of acceleration are:

$$a_x = x''(t) = -R\omega^2 \cos(\omega t); a_y = y''(t) = -R\omega^2 \sin(\omega t) .$$

Therefore, absolute value of acceleration is $a = \sqrt{a_x^2 + a_y^2} = \omega^2 R$, or using expression for velocity,

$$a = \frac{v^2}{R} .$$