An earth's satellite has a time period of90 min. Assuming the orbit to be circular, calculate its height. Given : G = 6.67 x 10-11 N-m2/kg2
, M = 6.02 x 1024kg and radius of the earth is
6400km.

Answer:

We are given:

$$T = 90 \text{ min} = 5400s$$

$$G = 6.67 * 10^{-11} \frac{Nm^2}{kg^2}$$

$$M = \ 6.02 * 10^{24}\ kg$$

$$R = 6400\ km = 6.4 * 10^6 \quad m$$

In any circular orbit, the centripetal force required to maintain the orbit (F_c) is provided by the gravitational force on the satellite (F_g). To calculate the geostationary orbit altitude, one begins with this equivalence:

$$F_c = F_g$$

By Newton's second law of motion, we can replace the forces **F** with the mass *m* of the object multiplied by the acceleration felt by the object due to that force:

$$ma_c = mg$$

We note that the mass of the satellite *m* appears on both ,so calculating the altitude simplifies into calculating the point where the magnitudes of the centripetal acceleration required for orbital motion and thegravitational acceleration provided by Earth's gravity are equal.

The centripetal acceleration's magnitude is:

$$|a_c| = \omega^2 r = \left(\frac{2\pi}{T}\right)^2 r$$

where ω is the angular speed, and *r* is the orbital radius as measured from the Earth's center of mass.

The magnitude of the gravitational acceleration is:

$$|g| = \frac{GM}{r^2}$$

where *M* is the mass of Earth, and *G* is the gravitational constant,

Equating the two accelerations gives:

$$r^3 = \frac{GM}{4\pi^2} T^2$$

$$r = \sqrt[3]{\frac{GM}{4\pi^2}T^2}$$

Height of the satellite is:

$$H = r - R = \sqrt[3]{\frac{GM}{4\pi^2}T^2} - R$$

Calculation:

$$H = \sqrt[3]{\frac{6.67 * 10^{-11} * \ 6.02 * 10^{24}}{4\pi^2}5400^2} - 6.4 * 10^6 \approx 2.69 * 10^5\ m = \mathbf{269\ km}$$

Answer: 269 km