

Suppose we are told the acceleration of a particle moving in a circle of a radius r with uniform speed v is proportional to some power of r say n , some power of v , say m . Determine the powers of r and v ?

Solution

We will use dimensional method.

$$\text{The dimension of } v \text{ is } [v] = \left[\frac{\text{meter}}{\text{second}} \right] = \left[\frac{m}{s} \right].$$

$$\text{The dimension of } r \text{ is } [r] = [\text{meter}] = [m].$$

$$\text{The dimension of acceleration is } [a] = \left[\frac{\text{meter}}{\text{second}^2} \right] = \left[\frac{m}{s^2} \right]$$

If $a \propto v^n r^m$, we have

$$[a] = [v^n r^m] = \left[\frac{m}{s} \right]^n [m]^m = \left[\frac{m}{s^2} \right]$$

From whence we get the following system of equations:

$$\begin{cases} n + m = 1 \\ n = 2 \end{cases} \Rightarrow$$

$$\begin{cases} m = -1 \\ n = 2 \end{cases} \Rightarrow$$

From whence (using dimensional method we don't know the coefficient of proportionality):

$$a \propto \frac{v^2}{r}.$$

Answer:

$$\begin{cases} m = -1 \\ n = 2 \end{cases}$$

$$a \propto \frac{v^2}{r}$$