Question 32548

Let z be the distance from center of the disc to the given point. Also, let the disc have radius r and surface charge density σ .

An infinitesimal potential created by charge dq is $d\varphi = \frac{k\,dq}{R}$, where $k = \frac{1}{4\pi\epsilon_0}$ is the Coulomb's constant and R is the distance from point of disc to the given point. In order to find the whole potential, one needs to integrate over the disc.

Elementary charge dq in terms of charge density is $dq = \sigma\, dS = \sigma\, 2\pi R'\, dR'$, where R' is the distance from the center of disc to the circle at disc which creates elementary potential (in the plane of disc). Distance R according to Pythagoras theorem is $R = \sqrt{R'^2 + z^2}$. Hence:

$$\varphi = 2\pi\sigma k \int_0^r \frac{R'\,dR'}{\sqrt{R'^2 + z^2}} = 2\pi k \sigma \sqrt{R'^2 + z^2}\,|_0^r = 2\pi\sigma k\left(\sqrt{r^2 + z^2} - z\right) .$$