Task. You are given 3 resistors each of $r = 3$ ohm and you are asked to get all possible values resistance when you connect them in different combinations. How many values of resistnce you can get?

Solution. Recall that if we have two resistors r_1 and r_2, then connecting them in series will result the resistance

$$r = r_1 + r_2,$$

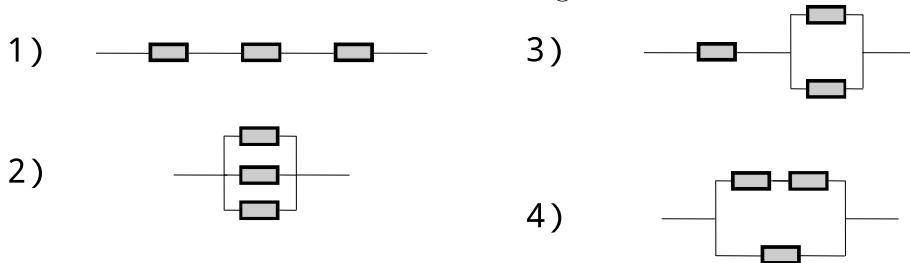and connecting them parallel will give the resistance r satisfying the identity:

$$\frac{1}{r} = \frac{1}{r_1} + \frac{1}{r_2},$$

whence

$$r = \frac{r_1 r_2}{r_1 + r_2}.$$

Notice that in these formulas changing the order of r_1 and r_2 does not change the resulting resistance.

Therefore we can connect 3 resistors in the following 4 distinct combinations:

1) All 3 resistors are in series. In this case the resulting resistance is

$$R = r + r + r = 3r = 3 * 3 \ ohm = 9 \ ohm.$$

2) All 3 resistors are parallel. In this case the resulting resistance satirsies

$$\frac{1}{R} = \frac{1}{r} + \frac{1}{r} + \frac{1}{r} = \frac{3}{r},$$

whence

$$R = \frac{r}{3} = \frac{3}{3} = 1 \ ohm.$$

3) Two resistors in series connected parallel with one resistor. In this case the resulting resistance satirsies

$$\frac{1}{R} = \frac{1}{r} + \frac{1}{r + r} = \frac{1}{r} + \frac{1}{2r} = \frac{2 + 1}{2r} = \frac{3}{2r},$$

whence

$$R = \frac{2r}{3} = \frac{2 * 3}{3} = 2 \ ohm.$$

4) One resistor connected with two parallel resistors. In this case the resulting resistance satirsies

$$R = r + \frac{r * r}{r + r} = r + \frac{r^2}{2r} = r + \frac{r}{2} = 1.5r = 1.5 * 3 = 4.5 \ ohm.$$

Answer. 1, 2, 4.5 and 9 ohms.